北京大学计算机学院

2025 年春季学期　　《编译原理(实验班)》

第十一讲　函数式语言的编译

Compilation of Functional Languages

主要内容

- 函数式编程语言 micro-ML
- 中间表示：若干种正规形式
- 前端到中端：源程序到正规形式的转换
- 中端到后端：正规形式程序的闭包转换
- 中端上的优化
- 参考教材：Peter Sestoft. 2017. Programming Language Concepts. Springer Cham. DOI: 10.1007/978-3-319-60789-4

主要内容

- **函数式编程语言 micro-ML**
- 中间表示：若干种正规形式
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表达式语言

- 让我们先从**表达式语言**开始

```
x - 11
```

```
(x - 11) * 2
```

x 是表达式中的自由变量(free variable)

```
if (x - 11) == 0 then y + 2 else y / 3
```

x 和 y 是表达式中的自由变量

```
let x = 7 in
  let y = 2 in
    if (x - 11) == 0 then y + 2 else y / 3
  end
end
```

let x = exp1 in exp2 end 是引入变量绑定的表达式，它首先计算 exp1 的值并绑定到 x 上，再计算并返回 exp2 的值

```
let x = 7 in
  let y = 2 in
    let y = let x = x - 1 in x + y end
    in x - y
    end
  end
end
```

let 表达式可以嵌套，并且可以引入重复的变量绑定，需要考虑变量的作用域问题

表达式语言的语法

- 本讲不考虑语法分析问题，直接讨论**抽象语法**

$$Exp ::= \text{const}(n) \mid \text{op}(Exp_1, Exp_2) \mid \text{if}(Exp_1, Exp_2, Exp_3) \mid \text{var}(x) \mid \text{let}(x, Exp_1, Exp_2)$$

```
(x - 11) * 2
```

→

```
mul(sub(var("x"), const(11)), const(2))
```

```
if (x - 11) == 0 then y + 2 else y / 3
```

```
if(eq(sub(var("x"), const(11)), const(0)), add(var("y"), const(2)), div(var("y"), const(3)))
```

```
let x = 7 in
  let y = 2 in
    if (x - 11) == 0 then y + 2 else y / 3
  end
end
```

→

```
let("x", const(7), let("y", const(2), ⋯))
```

表达式语言的解释求值

- **如何构造一个属性文法对表达式进行求值?**
- 对表达式求值需要知道其中所有**自由变量**的值
- **环境(environment)**:维护自由变量到值的绑定的数据结构
  - 用符号 ρ 来指代环境
  - 用 [] 表示空环境,即没有任何变量绑定
  - 用 $[var \mapsto val]\rho$ 表示在 ρ 上扩展 var 到 val 的绑定后得到的新环境
    - 允许有相同的变量绑定,后绑定的「覆盖」先绑定的
  - 用 $[var_1 \mapsto val_1, var_2 \mapsto val_2]\rho$ 表示 $[var_1 \mapsto val_1]([var_2 \mapsto val_2]\rho)$
  - 用 $[var_1 \mapsto val_1, var_2 \mapsto val_2, \cdots]$ 描述一个完整的环境

表达式语言的解释求值

$\rho = [x \mapsto 20]$

```
(x - 11) * 2
```

求值结果为 18

$\rho = [y \mapsto 2, x \mapsto 11]$

```
if (x - 11) == 0 then y + 2 else y / 3
```

求值结果为 4

初始为空环境

$\rho = []$

$\rho_1 = [x \mapsto 7]$

$\rho_2 = [y \mapsto 2, x \mapsto 7]$

```
let x = 7 in
  let y = 2 in
    let y = let x = x - 1 in x + y end
    in x - y
    end
  end
end
```

$\rho_3 = [x \mapsto 6, y \mapsto 2, x \mapsto 7]$

$\rho_4 = [y \mapsto 8, y \mapsto 2, x \mapsto 7]$

求值结果为 -1

表达式语言的解释求值

$$Exp ::= \text{const}(n) \mid \text{op}(Exp_1, Exp_2) \mid \text{if}(Exp_1, Exp_2, Exp_3) \mid \text{var}(x) \mid \text{let}(x, Exp_1, Exp_2)$$

综合属性 val 记录求值结果

继承属性 ρ 记录求值环境

产生规则	属性计算规则
$E \to \text{const}(n)$	$E.\text{val} = n$
$E \to \text{op}(E_1, E_2)$	$E_1.\rho = E.\rho$ $\quad E_2.\rho = E.\rho$ $E.\text{val} = f_{\text{op}}(E_1.\text{val}, E_2.\text{val})$
$E \to \text{if}(E_1, E_2, E_3)$	$E_1.\rho = E.\rho$ $\quad E_2.\rho = E.\rho$ $\quad E_3.\rho = E.\rho$ $E.\text{val} = E_1.\text{val}\ ?\ E_2.\text{val}\ :\ E_3.\text{val}$
$E \to \text{var}(x)$	$E.\text{val} = E.\rho(x)$
$E \to \text{let}(x, E_1, E_2)$	$E_1.\rho = E.\rho$ $\quad \boxed{E_2.\rho = [x \mapsto E_1.\text{val}]E.\rho}$ $E.\text{val} = E_2.\text{val}$

表达式语言的解释求值

- 为了方便后面讨论(比如递归)，使用**逻辑系统**表述求值规则
- 用 $\rho \vdash E \Rightarrow v$ 表示**三元关系** (ρ, E, v)，表示「在环境 ρ 下对表达式 E 进行求值的结果是 v」

横线上方表示「前提」

$$\frac{}{\rho \vdash \mathsf{const}(n) \Rightarrow n}$$

$$\frac{\rho \vdash E_1 \Rightarrow v_1 \quad \rho \vdash E_2 \Rightarrow v_2 \quad v = f_{\mathsf{op}}(v_1, v_2)}{\rho \vdash \mathsf{op}(E_1, E_2) \Rightarrow v}$$

横线下方表示「结论」

$$\frac{\rho \vdash E_1 \Rightarrow \mathsf{true} \quad \rho \vdash E_2 \Rightarrow v}{\rho \vdash \mathsf{if}(E_1, E_2, E_3) \Rightarrow v}$$

$$\frac{\rho \vdash E_1 \Rightarrow \mathsf{false} \quad \rho \vdash E_3 \Rightarrow v}{\rho \vdash \mathsf{if}(E_1, E_2, E_3) \Rightarrow v}$$

$$\frac{\rho(x) = v}{\rho \vdash \mathsf{var}(x) \Rightarrow v}$$

$$\frac{\rho \vdash E_1 \Rightarrow v_1 \qquad [x \mapsto v_1]\rho \vdash E_2 \Rightarrow v}{\rho \vdash \mathsf{let}(x, E_1, E_2) \Rightarrow v}$$

一阶函数

- 在表达式语言的基础上加入**一阶函数**，即函数的参数和返回值必须是标量值(比如布尔值、整型值)

```
let f x = x - 11 in f (f 77) end
```

定义了一个名为 f 的函数

```
let f x =
  let g y = x + y in g (2 * x) end
in
  f 7
end
```

定义了一个名为 f 的函数，其内部嵌套定义了一个名为 g 的函数，而 g 的函数体使用了 f 函数的参数

```
let f x =
  if x == 0 then 1 else 2 * f (x - 1)
in
  f y
end
```

定义了一个名为 f 的递归函数，整个表达式计算的返回值为 2^y

一阶函数的解释求值

$$Exp ::= \cdots \mid \text{letfun}(f, x, Exp_1, Exp_2) \mid \text{call}(f, Exp_1)$$

- **需要在环境中记录函数名到函数定义的映射**
  - 对于 $\text{letfun}(f, x, E_1, E_2)$，记录 $\text{fun}(f, x, E_1)$ 是否足够？

$\rho = [\text{f} \mapsto \text{fun(f, x, sub(var("x"), const(11)))}]$

```
let f x = x - 11 in f (f 77) end
```

求值结果为 55

$\rho = [\text{g} \mapsto \text{fun(g, y, add(var("x"), var("y")), x} \mapsto 7]$

```
let f x =
  let g y = x + y in g (2 * x) end
in
  f 7
end
```

对 g 的函数调用求值时，需要知道 g 被加入环境时的 x 的值

为什么需要这个限制？

一阶函数的解释求值

```
let f x =
  let g y = x + y in
    let x = 42 in g (2 * x) end
  end
in
  f 7
end
```

$\rho = [x \mapsto 42, g \mapsto \text{fun}(g, y, \text{add}(\text{var}("x"), \text{var}("y"))), x \mapsto 7]$

在本讲中，我们使用「静态」作用域（也称为「词法」作用域）

求值结果为 91

$$\frac{[f \mapsto \text{fun}(f, x, E_1)]\rho \vdash E_2 \Rightarrow v}{\rho \vdash \text{letfun}(f, x, E_1, E_2) \Rightarrow v}$$

这样做是否正确？

$$\frac{\rho \vdash E_1 \Rightarrow v_1 \qquad \boxed{\rho = [\cdots][f \mapsto \text{fun}(f', x', E_1')]\rho'} \qquad [x' \mapsto v_1]\rho' \vdash E_1' \Rightarrow v}{\rho \vdash \text{call}(f, E_1) \Rightarrow v}$$

一阶函数的解释求值

$$\frac{\rho \vdash E_1 \Rightarrow v_1 \qquad \rho = [\cdots][f \mapsto \text{fun}(f', x', E_1')]\rho' \qquad [x' \mapsto v_1]\rho' \vdash E_1' \Rightarrow v}{\rho \vdash \text{call}(f, E_1) \Rightarrow v}$$

```
let f x =
  if x == 0 then 1 else 2 * f (x - 1)
in
  f 2
end
```

$\rho_2 = [\text{x} \mapsto 2]$

环境中没有 f 的定义

$\rho_1 = [\text{f} \mapsto \text{fun}(\text{f}, \text{x}, \cdots)]$

为了支持递归调用，在环境中继续保留被调用函数的定义

$$\frac{\rho \vdash E_1 \Rightarrow v_1 \qquad \rho = [\cdots][f \mapsto \text{fun}(f', x', E_1')]\rho' \qquad [x' \mapsto v_1, f' \mapsto \cdots]\rho' \vdash E_1' \Rightarrow v}{\rho \vdash \text{call}(f, E_1) \Rightarrow v}$$

```
let f x =
  if x == 0 then 1 else 2 * f (x - 1)
in
  f 2
end
```

$\rho_2 = [\text{x} \mapsto 2, \text{f} \mapsto \text{fun}(\text{f}, \text{x}, \cdots)]$

$\rho_3 = [\text{x} \mapsto 1, \text{f} \mapsto \text{fun}(\text{f}, \text{x}, \cdots)]$

$\rho_4 = [\text{x} \mapsto 0, \text{f} \mapsto \text{fun}(\text{f}, \text{x}, \cdots)]$

求值结果为 4

高阶函数

- 函数的参数和返回值也可以是函数

```
let f x =
  let g y = x + y in g end
in
  (f 2) 3
end
```

返回一个函数，可以认为 f 是一个双参数函数(也称为「柯里化」)

第一次函数调用 f 2 返回一个「加二」函数，第二次函数调用计算结果为 5

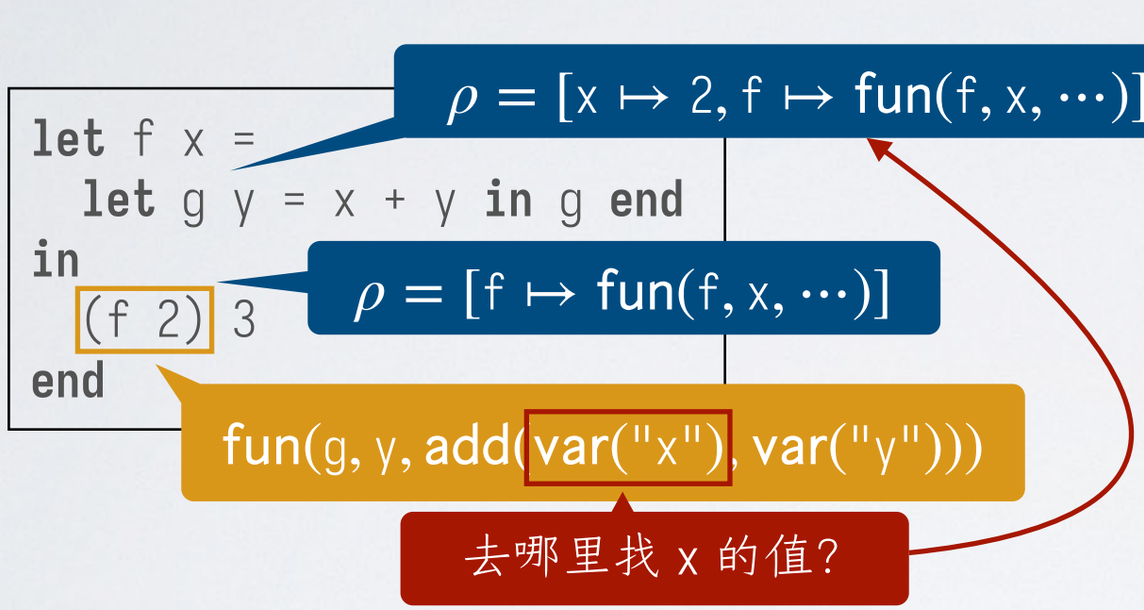
```
let f x =
  let g y = x + y in g end
in
  let h t = t 3 in
    h (f 2)
  end
end
```

参数 t 是一个函数，它接受整型值作为参数

函数调用 f 2 返回一个「加二」函数，并作为参数传递给 h 函数

高阶函数的解释求值

- **需要在环境中记录函数名到函数定义的映射**
  - 对于 letfun(f, x, E_1, E_2)，记录 fun(f, x, E_1) 是否足够？



- **闭包(closure)：在记录函数定义时，记录「当时」的环境**

$$\frac{[f \mapsto \text{clos}(f, x, E_1, \rho)]\rho \vdash E_2 \Rightarrow v}{\rho \vdash \text{letfun}(f, x, E_1, E_2) \Rightarrow v}$$

高阶函数的解释求值

$$\frac{\rho \vdash E_1 \Rightarrow v_1 \quad \boxed{\rho(f) = \mathtt{clos}(f', x', E_1', \rho')} \quad [x' \mapsto v_1, f' \mapsto \cdots]\rho' \vdash E_1' \Rightarrow v}{\rho \vdash \mathsf{call}(f, E_1) \Rightarrow v}$$

```
let f x =
  let g y = x + y in g end
in
  (f 2) 3
end
```

$\rho_2 = [\mathtt{x} \mapsto 2, \mathtt{f} \mapsto \mathtt{clos}(\mathtt{f}, \mathtt{x}, \cdots, [])]$

$\rho_1 = [\mathtt{f} \mapsto \mathtt{clos}(\mathtt{f}, \mathtt{x}, \cdots, [])]$

clos(g, y, add(var("x"), var("y")), ρ_2)

clos(g, y, add(var("x"), var("y")), ρ_2)

$\rho_3 = [\mathtt{y} \mapsto 3, \mathtt{g} \mapsto \cdots]\rho_2$

- 函数调用的函数 f 也可以是任意的表达式了：

$$\frac{\rho \vdash E_1 \Rightarrow v_1 \quad \boxed{\rho \vdash E_2 \Rightarrow \mathtt{clos}(f', x', E_1', \rho')} \quad [x' \mapsto v_1, f' \mapsto \cdots]\rho' \vdash E_1' \Rightarrow v}{\rho \vdash \mathsf{call}(E_2, E_1) \Rightarrow v}$$

高阶函数示例

- 下面这段代码在做什么计算？

$\rho_4 = [\texttt{x} \mapsto 0, \texttt{maker} \mapsto \textbf{clos}(\texttt{makefac}, \texttt{maker}, \cdots, [])]$

$\rho_3 = [\texttt{x} \mapsto 1, \texttt{maker} \mapsto \textbf{clos}(\texttt{makefac}, \texttt{maker}, \cdots, [])]$

$\rho_2 = [\texttt{x} \mapsto 2, \texttt{maker} \mapsto \textbf{clos}(\texttt{makefac}, \texttt{maker}, \cdots, [])]$

$\rho_1 = [\texttt{x} \mapsto 3, \texttt{maker} \mapsto \textbf{clos}(\texttt{makefac}, \texttt{maker}, \cdots, [])]$

```
let makefac maker =
  let f x = if x == 0 then 1 else x * ((maker maker) (x - 1))
  in f end
in
  let fac = makefac makefac
  in fac 3 end
end
```

$\textbf{clos}(\texttt{f}, \texttt{x}, \cdots, [\texttt{maker} \mapsto \textbf{clos}(\texttt{makefac}, \texttt{maker}, \cdots, [])])$

求值结果为 6

- 即便不原生支持递归函数，高阶函数也能「模拟」递归

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函数式语言的中间表示

- 由于**高阶函数**的存在，之前讲的三地址代码等 IR 都不合适
  - 或者说，在转换为三地址代码前还需要一种 IR
- 回顾：三地址代码主要简化了嵌套表达式的结构
- 作为表达式语言，函数式语言的 IR 也注重简化嵌套结构
  - K-Normal Form：运算分量、分支条件不嵌套，let 可以随意嵌套
  - A-Normal Form：运算分量、分支条件、let ... in 中间部分不嵌套
  - Continuation-Passing Style：运算分量分支条件不嵌套，只有尾递归
  - **从上往下，嵌套结构越简单，转换算法越复杂**

K-Normal Form

$$E ::= \text{const}(n) \mid \text{op}(E_1, E_2) \mid \text{if}(E_1, E_2, E_3) \mid \text{var}(x) \mid \text{let}(x, E_1, E_2) \mid \text{letfun}(f, x, E_1, E_2) \mid \text{call}(E_1, E_2)$$

- 运算分量、分支条件不嵌套，let 可以随意嵌套：

$$A ::= \text{const}(n) \mid \text{var}(x)$$

原子(atom)表达式

$$E ::= A \mid \text{op}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3) \mid \text{let}(x, E_1, E_2) \mid \text{letfun}(f, x, E_1, E_2) \mid \text{call}(A_1, A_2)$$

```
if (x - 11) == 0 then y + 2 else y / 3
```

→

```
let t1 = x - 11 in
  let t2 = t1 == 0 in
    if t2 then y + 2 else y / 3
  end
end
```

K-Normal Form

$$E ::= \text{const}(n) \mid \text{op}(E_1, E_2) \mid \text{if}(E_1, E_2, E_3) \mid \text{var}(x) \mid \text{let}(x, E_1, E_2) \mid \text{letfun}(f, x, E_1, E_2) \mid \text{call}(E_1, E_2)$$

- 运算分量、分支条件不嵌套，let 可以随意嵌套：

$$A ::= \text{const}(n) \mid \text{var}(x)$$

原子(atom)表达式

$$E ::= A \mid \text{op}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3) \mid \text{let}(x, E_1, E_2) \mid \text{letfun}(f, x, E_1, E_2) \mid \text{call}(A_1, A_2)$$

```
let f x =
  if x == 0 then 1 else 2 * f (x - 1)
in
  f (f 7)
end
```

```
let f x =
  let t1 = x == 0 in
    if t1 then 1 else
    let t2 = x - 1 in
      let t3 = f t2 in 2 * t3 end
    end
  end
in
  let t4 = f 7 in
    f t4
  end
end
```

K-Normal Form

$$E ::= \text{const}(n) \mid \text{op}(E_1, E_2) \mid \text{if}(E_1, E_2, E_3) \mid \text{var}(x) \mid \text{let}(x, E_1, E_2) \mid \text{letfun}(f, x, E_1, E_2) \mid \text{call}(E_1, E_2)$$

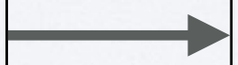
- 运算分量、分支条件不嵌套，let 可以随意嵌套：

$$A ::= \text{const}(n) \mid \text{var}(x)$$

原子(atom)表达式

$$E ::= A \mid \text{op}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3) \mid \text{let}(x, E_1, E_2) \mid \text{letfun}(f, x, E_1, E_2) \mid \text{call}(A_1, A_2)$$

```
let x = 7 in
  let y = 2 in
    let z =
      let w = x - 1 in w + y end
    in
      x - z
    end
  end
end
```



```
let x = 7 in
  let y = 2 in
    let z =
      let w = x - 1 in w + y end
    in
      x - z
    end
  end
end
```

A-Normal Form

$$E ::= \text{const}(n) \mid \text{op}(E_1, E_2) \mid \text{if}(E_1, E_2, E_3) \mid \text{var}(x) \mid \text{let}(x, E_1, E_2) \mid \text{letfun}(f, x, E_1, E_2) \mid \text{call}(E_1, E_2)$$

- 运算分量、分支条件、`let ... in` 中间部分不嵌套:

$$A ::= \text{const}(n) \mid \text{var}(x)$$
$$C ::= A \mid \text{op}(A_1, A_2) \mid \text{call}(A_1, A_2)$$
$$E ::= C \mid \text{if}(A_1, E_2, E_3) \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, x, E_1, E_2)$$

「复杂」表达式

```
let x = 7 in
  let y = 2 in
    let z =
      let w = x - 1 in w + y end
    in
      x - z
    end
  end
end
```

→

```
let x = 7 in
  let y = 2 in
    let w = x - 1 in
      let z = w + y in
        x - z
      end
    end
  end
end
```

A-Normal Form

$$E ::= \text{const}(n) \mid \text{op}(E_1, E_2) \mid \text{if}(E_1, E_2, E_3) \mid \text{var}(x) \mid \text{let}(x, E_1, E_2) \mid \text{letfun}(f, x, E_1, E_2) \mid \text{call}(E_1, E_2)$$

- 运算分量、分支条件、`let ... in` 中间部分不嵌套:

$$\begin{aligned} A &::= \text{const}(n) \mid \text{var}(x) \\ C &::= A \mid \text{op}(A_1, A_2) \mid \text{call}(A_1, A_2) \\ E &::= C \mid \text{if}(A_1, E_2, E_3) \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, x, E_1, E_2) \end{aligned}$$

```
let f x =
  let y =
    if x == 0 then 1 else x * f (x - 1)
  in
    (y + 2) / 5
  end
in
  f 7
end
```

分支表达式后面的代码会重复生成，可能造成代码大小指数增长

```
let f x =
  let t1 = x == 0 in
    if t1 then
    let y = 1 in let t2 = y + 2 in t2 / 5 end end
    else
    let t3 = x - 1 in
      let t4 = f t3 in
        let y = x * t4 in
          let t5 = y + 2 in t5 / 5 end end
        end end
  end
in f 7 end
```

A-Normal Form

$$E ::= \text{const}(n) \mid \text{op}(E_1, E_2) \mid \text{if}(E_1, E_2, E_3) \mid \text{var}(x) \mid \text{let}(x, E_1, E_2) \mid \text{letfun}(f, x, E_1, E_2) \mid \text{call}(E_1, E_2)$$

- 运算分量、分支条件、`let ... in` 中间部分不嵌套:

$$A ::= \text{const}(n) \mid \text{var}(x)$$
$$C ::= A \mid \text{op}(A_1, A_2) \mid \text{call}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3)$$
$$E ::= C \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, x, E_1, E_2)$$

允许 `if` 出现在 `let ... in` 中间部分

```
let f x =
  let y =
    if x == 0 then 1 else x * f (x - 1)
  in
    (y + 2) / 5
  end
in
  f 7
end
```

```
let f x =
  let t1 = x == 0 in
    let y =
      if t1 then 1 else
      let t2 = x - 1 in
        let t3 = f t3 in x * t3 end
      end
    in
      let t4 = y + 2 in t4 / 5 end
    end
  end
in f 7 end
```

Continuation-Passing Style

- 「continuation」的概念比较难解释，本讲中考虑如下约束：
- **任何的函数调用都不会(或者说不需要)返回其调用者**
- 如何实现这个效果？

每个函数接受一个回调(callback)来模拟函数返回

```
function id(x) {
  return x ;
}
```

```
function id(x, ret) {
  ret(x);
}
```

```
function fac(n) {
  if (n == 0)
    return 1;
  else
    return n * fac(n - 1);
}
```

```
function fac(n, ret) {
  if (n == 0)
    ret(1);
  else
    fac(n - 1, function (t1) { ret(n * t1) });
}
```

构造一个新的回调

Continuation-Passing Style

```
function fac(n) {
  return tail_fac(n, 1);
}

function tail_fac(n, a) {
  if (n == 0)
    return a;
  else
    return tail_fac(n - 1, n * a);
}
```

尾递归实现的阶乘

```
function fac(n, ret) {
  tail_fac(n, 1, ret);
}

function tail_fac(n, a, ret) {
  if (n == 0)
    ret(a);
  else
    tail_fac(n - 1, n * a, ret);
}
```

尾递归转换后不引入新回调

```
function fib(n) {
  if (n < 2)
    return 1;
  else
    return fib(n - 1) + fib(n - 2);
}
```

```
function fib(n, ret) {
  if (n < 2)
    ret(1);
  else
    fib(n - 1,
      function (t1) {
        fib(n - 2, function (t2) { ret(t1 + t2) })
      });
}
```

第一处递归的回调

第二处递归的回调

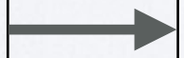
CPS 作为一种编程方式

- 这种编程方式称为 **Continuation-Passing Style (CPS)**
- 在异步编程中其实挺常见的，通过回调来实现**非阻塞**计算
  - fetch(url) 同步调用，会阻塞当前计算直到网络访问返回
  - fetch(url, ret) 异步调用，不阻塞计算，返回后由 ret 回调处理

- 例如，考虑组合数计算函数，其中的阶乘是服务器远程计算：

```
function comb(n, k) {
  return fac(n) / (fac(k) * fac(n - k));
}

function fac(n) {
  var res = fetch("./fact/" + n);
  return eval(res);
}
```



```
function comb(n, k, ret) {
  fac(n, function (fn) {
    fac(k, function (fk) {
      fac(n - k, function (fnk) {
        ret(fn / (fk * fnk))
      })
    })
  });
}

function fac(n, ret) {
  fetch("./fact/" + n, function (res) { ret(eval(res)) });
}
```

CPS 作为一种编程方式

- 如果用 CPS 风格写程序，需要改变**异常处理**等控制流的实现

```
function fac(n) {
  if (n < 0)
    throw "n < 0";
  else if (n == 0)
    return 1;
  else
  return n * fac(n - 1);
}

function total_fac(n) {
  try {
    return fac(n);
  } catch (ex) {
    return false;
  }
}
```

可以有多个回调函数，这里的 thro 回调用于进行异常处理

```
function fac(n, ret, thro) {
  if (n < 0)
    thro("n < 0");
  else if (n == 0)
    ret(1);
  else
    fac(n - 1, function (t1) { ret(n * t1) }, thro);
}

function total_fac(n, ret) {
  fac(n, ret, function (ex) { ret(false) });
}
```

进行异常处理的回调

CPS 作为一种中间表示

$$E ::= \text{const}(n) \mid \text{op}(E_1, E_2) \mid \text{if}(E_1, E_2, E_3) \mid \text{var}(x) \mid \text{let}(x, E_1, E_2) \mid \text{letfun}(f, x, E_1, E_2) \mid \text{call}(E_1, E_2)$$

- 运算分量、分支条件不嵌套，只有尾递归，**每个函数加上回调参数**：

$$A ::= \text{const}(n) \mid \text{var}(x)$$

$$C ::= A \mid \text{op}(A_1, A_2)$$

$$E ::= C \mid \text{if}(A_1, E_2, E_3) \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, \overrightarrow{x_1, \cdots, x_n}, E_1, E_2) \mid \text{call}(A_0, \overrightarrow{A_1, \cdots, A_n})$$

```
let f x =
  if x == 0 then 1 else 2 * f (x - 1)
in
  f 7
end
```

```
let f x ret1 =
  let t1 = x == 0 in
    if t1 then ret1(1)
    else
    let t2 = x - 1 in
      let ret2 t3 = let t4 = 2 * t3 in ret1(t4) end
      in f t2 ret2 end
    end end
in
  let ret3 t5 = t5 in f 7 ret3 end
end
```

CPS 作为一种中间表示

$$E ::= \text{const}(n) \mid \text{op}(E_1, E_2) \mid \text{if}(E_1, E_2, E_3) \mid \text{var}(x) \mid \text{let}(x, E_1, E_2) \mid \text{letfun}(f, x, E_1, E_2) \mid \text{call}(E_1, E_2)$$

- 运算分量、分支条件不嵌套，只有尾递归，**每个函数加上回调参数**：

$$A ::= \text{const}(n) \mid \text{var}(x)$$

$$C ::= A \mid \text{op}(A_1, A_2)$$

$$E ::= C \mid \text{if}(A_1, E_2, E_3) \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, \overrightarrow{x_1, \cdots, x_n}, E_1, E_2) \mid \text{call}(A_0, \overrightarrow{A_1, \cdots, A_n})$$

```
let f x =
  let y =
    if x == 0 then 1 else x * f (x - 1)
  in
    (y + 2) / 5
  end
in
  ...
end
```

分支表达式后面的代码会重复生成，可能造成代码大小指数增长

```
let f x ret1 =
  let t1 = x == 0 in
    if t1 then
    let y = 1 in let t2 = y + 2 in let t3 = t2 / 5 in
      ret1(t3) end end end
    else
    let t4 = x - 1 in
      let ret2 t5 = let y = x * t5 in
        let t6 = y + 2 in let t7 = t6 / 5 in
          ret1(t7) end end end
      in f t4 ret2 end
    end end in ... end
```

CPS 作为一种中间表示

$$E ::= \text{const}(n) \mid \text{op}(E_1, E_2) \mid \text{if}(E_1, E_2, E_3) \mid \text{var}(x) \mid \text{let}(x, E_1, E_2) \mid \text{letfun}(f, x, E_1, E_2) \mid \text{call}(E_1, E_2)$$

- 运算分量、分支条件不嵌套，只有尾递归，**每个函数加上回调参数**

```
let f x =
  let y =
    if x == 0 then 1 else x * f (x - 1)
  in
    (y + 2) / 5
  end
in
  ...
end
```

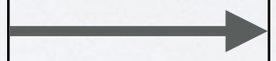
为分支表达式的后续计算引入一个新的回调

```
let f x ret1 =
  let t1 = x == 0 in
    let retif y =
      let t2 = y + 2 in let t3 = t2 / 5 in
        ret1(t3) end end
    in
      if t1 then retif(1)
      else
      let t4 = x - 1 in
        let ret2 t5 =
          let y = x * t5 in retif(y) end
        in f t4 ret2 end
      end
    end
  end
in ... end
```

CPS 与 SSA

- 尾递归函数调用就像是控制流图上的 goto 语句
- 对于一阶函数，可以改写为支持带参数 goto 的Koopa IR 形式

```
let f x ret1 =
  let t1 = x == 0 in
    let retif y =
      let t2 = y + 2 in let t3 = t2 / 5 in
        ret1(t3) end end
    in
      if t1 then retif(1)
      else
      let t4 = x - 1 in
        let ret2 t5 =
          let y = x * t5 in retif(y) end
        in f t4 ret2 end
      end
    end
  end
in ... end
```



```
f(x):
  t1 = x == 0
  if t1 then
    goto retif(1)
  else
    t4 = x - 1
    t5 = f(t4)
    y = x * t5
    goto retif(y)

retif(y):
  t2 = y + 2
  t3 = t2 / 5
  return t3
```

主要内容

- 函数式编程语言 micro-ML
- 中间表示：若干种正规形式
- **前端到中端：源程序到正规形式的转换**
- 中端到后端：正规形式程序的闭包转换
- 中端上的优化

转换为 K-Normal Form (KNF)

$$E ::= \text{const}(n) \mid \text{op}(E_1, E_2) \mid \text{if}(E_1, E_2, E_3) \mid \text{var}(x) \mid \text{let}(x, E_1, E_2) \mid \text{letfun}(f, x, E_1, E_2) \mid \text{call}(E_1, E_2)$$

$$A ::= \text{const}(n) \mid \text{var}(x)$$
$$E ::= A \mid \text{op}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3) \mid \text{let}(x, E_1, E_2) \mid \text{letfun}(f, x, E_1, E_2) \mid \text{call}(A_1, A_2)$$

- 设计一个**属性文法**，把函数式程序转换为 KNF 形式

产生规则	属性计算规则
$E \to \text{const}(n)$	$E.\text{knf} = \text{const}(n)$
$E \to \text{op}(E_1, E_2)$	$E.\text{knf} = \text{let}(t_1, E_1.\text{knf}, \text{let}(t_2, E_2.\text{knf}, \text{op}(\text{var}(t_1), \text{var}(t_2))))$
$E \to \text{if}(E_1, E_2, E_3)$	$E.\text{knf} = \text{let}(t_1, E_1.\text{knf}, \text{if}(\text{var}(t_1), E_2.\text{knf}, E_3.\text{knf}))$
$E \to \text{var}(x)$	$E.\text{knf} = \text{var}(x)$
$E \to \text{let}(x, E_1, E_2)$	$E.\text{knf} = \text{let}(x, E_1.\text{knf}, E_2.\text{knf})$
$E \to \text{letfun}(f, x, E_1, E_2)$	$E.\text{knf} = \text{letfun}(f, x, E_1.\text{knf}, E_2.\text{knf})$
$E \to \text{call}(E_1, E_2)$	$E.\text{knf} = \text{let}(t_1, E_1.\text{knf}, \text{let}(t_2, E_2.\text{knf}, \text{call}(\text{var}(t_1), \text{var}(t_2))))$

生成临时变量名

转换为 A-Normal Form (ANF)

$$E ::= \text{const}(n) \mid \text{op}(E_1, E_2) \mid \text{if}(E_1, E_2, E_3) \mid \text{var}(x) \mid \text{let}(x, E_1, E_2) \mid \text{letfun}(f, x, E_1, E_2) \mid \text{call}(E_1, E_2)$$

$$A ::= \text{const}(n) \mid \text{var}(x)$$
$$C ::= A \mid \text{op}(A_1, A_2) \mid \text{call}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3)$$
$$E ::= C \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, x, E_1, E_2)$$

- 先不考虑函数，主要考虑如何将嵌套 let 表达式「序列化」：

```
let x = 7 in
  let y = 2 in
    let z =
      let w = x - 1 in w + y end
    in
      x - z
    end
  end
end
```

处理 let z = ... in 中间的表达式时，要考虑上下文的情况，所以需要某种继承属性

```
let x = 7 in
  let y = 2 in
    let w = x - 1 in
      let z = w + y in
        x - z
      end
    end
  end
end
```

转换为 A-Normal Form (ANF)

```
function normalize(e, k) {
  if (e.kind == "let") {
    return normalize(e.e1, function (c1) {
      return exp("let", e.x, c1, normalize(e.e2, k));
    });
  }
  else if (e.kind == "var" || e.kind == "const")
    return k(e);
  else if (e.kind == "op") {
    return normalize(e.e1, function (c1) {
      var t1 = gentmp();
      return normalize(e.e2, function (c2) {
        var t2 = gentmp();
        return exp("let", t1, c1,
                   exp("let", t2, c2,
                   k(exp("op", exp("var", t1), exp("var", t2)))));
      });
    });
  } ...
}
```

继承属性是一个「函数」，接受一个表达式 C 表示 e 转换后（可能已经生成了若干层 let）表示其结果的表达式，返回在整个转换上下文中得到的表达式 E

当 c1 是原子表达式时，不用生成 let 绑定

```
k1 = fun (c) { return c; }
```

$$A ::= \text{const}(n) \mid \text{var}(x)$$
$$C ::= A \mid \text{op}(A_1, A_2) \mid \text{call}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3)$$
$$E ::= C \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, x, E_1, E_2)$$

```
let x = 7 in
  let y = 2 in
    let z =
      let w = x - 1 in w + y end
    in
      x - z
    end
  end
end
```

转换为 A-Normal Form (ANF)

```
function normalize(e, k) {
  if (e.kind == "let") {
    return normalize(e.e1, function (c1) {
      return exp("let", e.x, c1, normalize(e.e2, k));
    });
  }
  else if (e.kind == "var" || e.kind == "const")
    return k(e);
  else if (e.kind == "op") {
    return normalize(e.e1, function (c1) {
      var t1 = gentmp();
      return normalize(e.e2, function (c2) {
        var t2 = gentmp();
        return exp("let", t1, c1,
                   exp("let", t2, c2,
                   k(exp("op", exp("var", t1), exp("var", t2)))));
      });
    });
  } ...
}
```

继承属性是一个「函数」，接受一个表达式 C 表示 e 转换后（可能已经生成了若干层 let）表示其结果的表达式，返回在整个转换上下文中得到的表达式 E

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$$E ::= C \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, x, E_1, E_2)$$

```
let x = 7 in
  let y = 2 in
    let z =
      let w = x - 1 in w + y end
    in
      x - z
    end
  end
end
```

```
k1 = fun (c) { return c; }
k2 = fun (c1) { return let("x", c1, norm("let y ...", k1)); }
```

转换为 A-Normal Form (ANF)

```
function normalize(e, k) {
  if (e.kind == "let") {
    return normalize(e.e1, function (c1) {
      return exp("let", e.x, c1, normalize(e.e2, k));
    });
  }
  else if (e.kind == "var" || e.kind == "const")
    return k(e);
  else if (e.kind == "op") {
    return normalize(e.e1, function (c1) {
      var t1 = gentmp();
      return normalize(e.e2, function (c2) {
        var t2 = gentmp();
        return exp("let", t1, c1,
                   exp("let", t2, c2,
                   k(exp("op", exp("var", t1), exp("var", t2)))));
      });
    });
  } ...
}
```

继承属性是一个「函数」，接受一个表达式 C 表示 e 转换后（可能已经生成了若干层 let）表示其结果的表达式，返回在整个转换上下文中得到的表达式 E

当 c1 是原子表达式时，不用生成 let 绑定

```
k1 = fun (c) { return c; }
```

$$A ::= \text{const}(n) \mid \text{var}(x)$$
$$C ::= A \mid \text{op}(A_1, A_2) \mid \text{call}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3)$$
$$E ::= C \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, x, E_1, E_2)$$

```
let x = 7 in
  let y = 2 in
    let z =
      let w = x - 1 in w + y end
    in
      x - z
    end
  end
end
```

转换为 A-Normal Form (ANF)

```
function normalize(e, k) {
  if (e.kind == "let") {
    return normalize(e.e1, function (c1) {
      return exp("let", e.x, c1, normalize(e.e2, k));
    });
  }
  else if (e.kind == "var" || e.kind == "const")
    return k(e);
  else if (e.kind == "op") {
    return normalize(e.e1, function (c1) {
      var t1 = gentmp();
      return normalize(e.e2, function (c2) {
        var t2 = gentmp();
        return exp("let", t1, c1,
                   exp("let", t2, c2,
                   k(exp("op", exp("var", t1), exp("var", t2)))));
      });
    });
  } ...
}
```

继承属性是一个「函数」，接受一个表达式 C 表示 e 转换后（可能已经生成了若干层 let）表示其结果的表达式，返回在整个转换上下文中得到的表达式 E

当 c1 是原子表达式时，不用生成 let 绑定

$$A ::= \text{const}(n) \mid \text{var}(x)$$
$$C ::= A \mid \text{op}(A_1, A_2) \mid \text{call}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3)$$
$$E ::= C \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, x, E_1, E_2)$$

```
let x = 7 in
  let y = 2 in
    let z =
      let w = x - 1 in w + y end
    in
      x - z
    end
  end
end
```

```
k1 = fun (c) { return c; }
```

```
k3 = fun (c1) { return let("y", c1, norm("let z ...", k1)); }
```

转换为 A-Normal Form (ANF)

```
function normalize(e, k) {
  if (e.kind == "let") {
    return normalize(e.e1, function (c1) {
      return exp("let", e.x, c1, normalize(e.e2, k));
    });
  }
  else if (e.kind == "var" || e.kind == "const")
    return k(e);
  else if (e.kind == "op") {
    return normalize(e.e1, function (c1) {
      var t1 = gentmp();
      return normalize(e.e2, function (c2) {
        var t2 = gentmp();
        return exp("let", t1, c1,
                   exp("let", t2, c2,
                   k(exp("op", exp("var", t1), exp("var", t2)))));
      });
    });
  } ...
}
```

继承属性是一个「函数」，接受一个表达式 C 表示 e 转换后（可能已经生成了若干层 let）表示其结果的表达式，返回在整个转换上下文中得到的表达式 E

当 c1 是原子表达式时，不用生成 let 绑定

```
k1 = fun (c) { return c; }
```

$$A ::= \text{const}(n) \mid \text{var}(x)$$
$$C ::= A \mid \text{op}(A_1, A_2) \mid \text{call}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3)$$
$$E ::= C \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, x, E_1, E_2)$$

```
let x = 7 in
  let y = 2 in
    let z =
      let w = x - 1 in w + y end
    in
      x - z
    end
  end
end
```

转换为 A-Normal Form (ANF)

```
function normalize(e, k) {
  if (e.kind == "let") {
    return normalize(e.e1, function (c1) {
      return exp("let", e.x, c1, normalize(e.e2, k));
    });
  }
  else if (e.kind == "var" || e.kind == "const")
    return k(e);
  else if (e.kind == "op") {
    return normalize(e.e1, function (c1) {
      var t1 = gentmp();
      return normalize(e.e2, function (c2) {
        var t2 = gentmp();
        return exp("let", t1, c1,
                   exp("let", t2, c2,
                   k(exp("op", exp("var", t1), exp("var", t2)))));
      });
    });
  } ...
}
```

继承属性是一个「函数」，接受一个表达式 C 表示 e 转换后（可能已经生成了若干层 let）表示其结果的表达式，返回在整个转换上下文中得到的表达式 E

当 c1 是原子表达式时，不用生成 let 绑定

$$A ::= \text{const}(n) \mid \text{var}(x)$$
$$C ::= A \mid \text{op}(A_1, A_2) \mid \text{call}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3)$$
$$E ::= C \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, x, E_1, E_2)$$

```
let x = 7 in
  let y = 2 in
    let z =
      let w = x - 1 in w + y end
    in
      x - z
    end
  end
end
```

```
k1 = fun (c) { return c; }
```

```
k4 = fun (c1) { return let("z", c1, norm("x - z", k1)); }
```

转换为 A-Normal Form (ANF)

```
function normalize(e, k) {
  if (e.kind == "let") {
    return normalize(e.e1, function (c1) {
      return exp("let", e.x, c1, normalize(e.e2, k));
    });
  }
  else if (e.kind == "var" || e.kind == "const")
    return k(e);
  else if (e.kind == "op") {
    return normalize(e.e1, function (c1) {
      var t1 = gentmp();
      return normalize(e.e2, function (c2) {
        var t2 = gentmp();
        return exp("let", t1, c1,
                   exp("let", t2, c2,
                   k(exp("op", exp("var", t1), exp("var", t2)))));
      });
    });
  } ...
}
```

继承属性是一个「函数」，接受一个表达式 C 表示 e 转换后（可能已经生成了若干层 let）表示其结果的表达式，返回在整个转换上下文中得到的表达式 E

当 c1 是原子表达式时，不用生成 let 绑定

$$A ::= \text{const}(n) \mid \text{var}(x)$$
$$C ::= A \mid \text{op}(A_1, A_2) \mid \text{call}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3)$$
$$E ::= C \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, x, E_1, E_2)$$

```
let x = 7 in
  let y = 2 in
    let z =
      let w = x - 1 in w + y end
    in
      x - z
    end
  end
end
```

```
k1 = fun (c) { return c; }
k4 = fun (c1) { return let("z", c1, norm("x - z", k1)); }
k5 = fun (c1) { return let("w", c1, norm("w + y", k4)); }
```

转换为 A-Normal Form (ANF)

```
function normalize(e, k) {
  if (e.kind == "let") {
    return normalize(e.e1, function (c1) {
      return exp("let", e.x, c1, normalize(e.e2, k));
    });
  }
  else if (e.kind == "var" || e.kind == "const")
    return k(e);
  else if (e.kind == "op") {
    return normalize(e.e1, function (c1) {
      var t1 = gentmp();
      return normalize(e.e2, function (c2) {
        var t2 = gentmp();
        return exp("let", t1, c1,
                   exp("let", t2, c2,
                   k(exp("op", exp("var", t1), exp("var", t2)))));
      });
    });
  } ...
}
```

继承属性是一个「函数」，接受一个表达式 C 表示 e 转换后（可能已经生成了若干层 let）表示其结果的表达式，返回在整个转换上下文中得到的表达式 E

当 c1 是原子表达式时，不用生成 let 绑定

$$A ::= \text{const}(n) \mid \text{var}(x)$$
$$C ::= A \mid \text{op}(A_1, A_2) \mid \text{call}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3)$$
$$E ::= C \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, x, E_1, E_2)$$

```
let x = 7 in
  let y = 2 in
    let z =
      let w = x - 1 in w + y end
    in
      x - z
    end
  end
end
```

```
k1 = fun (c) { return c; }
```

```
k4 = fun (c1) { return let("z", c1, norm("x - z", k1)); }
```

转换为 A-Normal Form (ANF)

```
function normalize(e, k) {
  if (e.kind == "let") {
    return normalize(e.e1, function (c1) {
      return exp("let", e.x, c1, normalize(e.e2, k));
    });
  }
  else if (e.kind == "var" || e.kind == "const")
    return k(e);
  else if (e.kind == "op") {
    return normalize(e.e1, function (c1) {
      var t1 = gentmp();
      return normalize(e.e2, function (c2) {
        var t2 = gentmp();
        return exp("let", t1, c1,
                   exp("let", t2, c2,
                   k(exp("op", exp("var", t1), exp("var", t2)))));
      });
    });
  } ...
}
```

继承属性是一个「函数」，接受一个表达式 C 表示 e 转换后（可能已经生成了若干层 let）表示其结果的表达式，返回在整个转换上下文中得到的表达式 E

当 c1 是原子表达式时，不用生成 let 绑定

```
k1 = fun (c) { return c; }
```

$$A ::= \text{const}(n) \mid \text{var}(x)$$
$$C ::= A \mid \text{op}(A_1, A_2) \mid \text{call}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3)$$
$$E ::= C \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, x, E_1, E_2)$$

```
let x = 7 in
  let y = 2 in
    let w = x - 1 in
      let z = w + y in
        x - z
      end
    end
  end
end
```

转换为 A-Normal Form (ANF)

```
function normalize(e, k) {
  if (e.kind == "let") {
    return normalize(e.e1, function (c1) {
      return exp("let", e.x, c1, normalize(e.e2, k));
    });
  }
  else if (e.kind == "var" || e.kind == "const")
    return k(e);
  else if (e.kind == "op") {
    return normalize(e.e1, function (c1) {
      var t1 = gentmp();
      return normalize(e.e2, function (c2) {
        var t2 = gentmp();
        return exp("let", t1, c1,
                   exp("let", t2, c2,
                   k(exp("op", exp("var", t1), exp("var", t2)))));
      });
    });
  } ...
}
```

继承属性是一个「函数」，接受一个表达式 C 表示 e 转换后（可能已经生成了若干层 let）表示其结果的表达式，返回在整个转换上下文中得到的表达式 E

当 c1 是原子表达式时，不用生成 let 绑定

$$A ::= \text{const}(n) \mid \text{var}(x)$$
$$C ::= A \mid \text{op}(A_1, A_2) \mid \text{call}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3)$$
$$E ::= C \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, x, E_1, E_2)$$

```
let x = 7 in
  let y = 2 in
    let w = x - 1 in
      let z = w + y in
        x - z
      end
    end
  end
end
```

转换为 A-Normal Form (ANF)

$$
\begin{aligned}
A &::= \text{const}(n) \mid \text{var}(x) \\
C &::= A \mid \text{op}(A_1, A_2) \mid \text{call}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3) \\
E &::= C \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, x, E_1, E_2)
\end{aligned}
$$

```
else if (e.kind == "if") {
  return normalize(e.e1, function (c1) {
    var t1 = gentmp();
    return exp("let", t1, c1,
               k(exp("if", exp("var", t1),
                     normalize(e.e2, function (c) { return c; }),
                     normalize(e.e3, function (c) { return c; }))));
  })
}
else if (e.kind == "letfun") {
  return exp("let", e.f, e.x,
             normalize(e.e1, function (c) { return c; }),
             normalize(e.e2, k));
}
else if (e.kind == "call") {
  ... // similiar to the op case
}
```

if 是 C 型表达式，可以作为 k 的参数

如果在这里使用 k，可能导致代码大小指数增长

对函数体定义转换时不用考虑 k

转换为 A-Normal Form (ANF)

```
let f x =
  let y =
    if x == 0 then 1 else x * f (x - 1)
  in
    (y + 2) / 5
  end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let y =
    if x == 0 then 1 else x * f (x - 1)
  in
    (y + 2) / 5
  end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

```
k2 = fun (c) { return c; }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let y =
    if x == 0 then 1 else x * f (x - 1)
  in
    (y + 2) / 5
  end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

```
k2 = fun (c) { return c; }
```

```
k3 = fun (c1) { return let("y", c1, norm("(y + 2) / 5", k2)); }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let y =
    if x == 0 then 1 else x * f (x - 1)
  in
    (y + 2) / 5
  end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

```
k2 = fun (c) { return c; }
```

```
k3 = fun (c1) { return let("y", c1, norm("(y + 2) / 5", k2)); }
```

```
k4 = fun (c1) { t1=gentmp(); return let(t1, c1, k3(if(t1, norm("1", id), norm("x * f(x-1)", id)))); }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let y =
    let t1 = x == 0 in
      if t1 then 1 else x * f (x - 1)
    end
  in
    (y + 2) / 5
  end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

```
k2 = fun (c) { return c; }
```

```
k3 = fun (c1) { return let("y", c1, norm("(y + 2) / 5", k2)); }
```

```
k5 = fun (c) { return c; }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let y =
    let t1 = x == 0 in
      if t1 then 1 else x * f (x - 1)
    end
  in
    (y + 2) / 5
  end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

```
k2 = fun (c) { return c; }
```

```
k3 = fun (c1) { return let("y", c1, norm("(y + 2) / 5", k2)); }
```

```
k6 = fun (c) { return c; }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let y =
    let t1 = x == 0 in
      if t1 then 1 else x * f (x - 1)
    end
  in
    (y + 2) / 5
  end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

```
k2 = fun (c) { return c; }
```

```
k3 = fun (c1) { return let("y", c1, norm("(y + 2) / 5", k2)); }
```

```
k6 = fun (c) { return c; }
```

```
k7 = fun (c1) { return norm("f(x-1)", fun (c2) { t2=gentmp(); return let(t2, c2, k6(c1 * t2)); }); }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let y =
    let t1 = x == 0 in
      if t1 then 1 else x * f (x - 1)
    end
  in
    (y + 2) / 5
  end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

```
k2 = fun (c) { return c; }
```

```
k3 = fun (c1) { return let("y", c1, norm("(y + 2) / 5", k2)); }
```

```
k6 = fun (c) { return c; }
```

```
k8 = fun (c2) { t2=gentmp(); return let(t2, c2, k6("x" * t2)); }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let y =
    let t1 = x == 0 in
      if t1 then 1 else x * f (x - 1)
    end
  in
    (y + 2) / 5
  end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

```
k2 = fun (c) { return c; }
```

```
k3 = fun (c1) { return let("y", c1, norm("(y + 2) / 5", k2)); }
```

```
k6 = fun (c) { return c; }
```

```
k8 = fun (c2) { t2=gentmp(); return let(t2, c2, k6("x" * t2)); }
```

```
k9 = fun (c1) { return norm("x-1", fun (c2) { t2=gentmp(); return let(t2, c2, k8(c1 t2)); }); }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let y =
    let t1 = x == 0 in
      if t1 then 1 else x * f (x - 1)
    end
  in
    (y + 2) / 5
  end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

```
k2 = fun (c) { return c; }
```

```
k3 = fun (c1) { return let("y", c1, norm("(y + 2) / 5", k2)); }
```

```
k6 = fun (c) { return c; }
```

```
k8 = fun (c2) { t2=gentmp(); return let(t2, c2, k6("x" * t2)); }
```

```
k10 = fun (c2) { t2=gentmp(); return let(t2, c2, k8("f" t2)); }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let y =
    let t1 = x == 0 in
      if t1 then 1 else
      let t2 = x - 1 in x * f t2 end
    end
  in
    (y + 2) / 5
  end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

```
k2 = fun (c) { return c; }
```

```
k3 = fun (c1) { return let("y", c1, norm("(y + 2) / 5", k2)); }
```

```
k6 = fun (c) { return c; }
```

```
k8 = fun (c2) { t2=gentmp(); return let(t2, c2, k6("x" * t2)); }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let y =
    let t1 = x == 0 in
      if t1 then 1 else
      let t2 = x - 1 in let t3 = f t2 in x * t3 end end
    end
  in
    (y + 2) / 5
  end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

```
k2 = fun (c) { return c; }
```

```
k3 = fun (c1) { return let("y", c1, norm("(y + 2) / 5", k2)); }
```

```
k6 = fun (c) { return c; }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let y =
    let t1 = x == 0 in
      if t1 then 1 else
      let t2 = x - 1 in let t3 = f t2 in x * t3 end end
    end
  in
    (y + 2) / 5
  end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

```
k2 = fun (c) { return c; }
```

```
k3 = fun (c1) { return let("y", c1, norm("(y + 2) / 5", k2)); }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let t1 = x == 0 in
    let y =
      if t1 then 1 else
      let t2 = x - 1 in let t3 = f t2 in x * t3 end end
    in
      (y + 2) / 5
    end
  end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

```
k2 = fun (c) { return c; }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let t1 = x == 0 in
    let y =
      if t1 then 1 else
      let t2 = x - 1 in let t3 = f t2 in x * t3 end end
    in
      (y + 2) / 5
    end
  end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

```
k2 = fun (c) { return c; }
```

```
k11 = fun (c1) { t1=gentmp(); return norm("5", fun (c2) { return k2(t1 / c2); }); }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let t1 = x == 0 in
    let y =
      if t1 then 1 else
      let t2 = x - 1 in let t3 = f t2 in x * t3 end end
    in
      let t4 = y + 2 in t4 / 5 end
    end
  end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

```
k2 = fun (c) { return c; }
```

```
k12 = fun (c2) { return k2(t1 / c2); }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let t1 = x == 0 in
    let y =
      if t1 then 1 else
      let t2 = x - 1 in let t3 = f t2 in x * t3 end end
    in
      let t4 = y + 2 in t4 / 5 end
    end
end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

```
k2 = fun (c) { return c; }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let t1 = x == 0 in
    let y =
      if t1 then 1 else
      let t2 = x - 1 in let t3 = f t2 in x * t3 end end
    in
      let t4 = y + 2 in t4 / 5 end
    end
  end
in
  f 7
end
```

```
k1 = fun (c) { return c; }
```

转换为 A-Normal Form (ANF)

```
let f x =
  let t1 = x == 0 in
    let y =
      if t1 then 1 else
      let t2 = x - 1 in let t3 = f t2 in x * t3 end end
    in
      let t4 = y + 2 in t4 / 5 end
    end
  end
in
  f 7
end
```

- 练习：设计一个**属性文法**，把函数式程序转换为 ANF 形式
- **挑战：如何把函数式程序转换为 CPS 形式？**

主要内容

- 函数式编程语言 micro-ML
- 中间表示：若干种正规形式
- 前端到中端：源程序到正规形式的转换
- **中端到后端：正规形式程序的闭包转换**
- 中端上的优化

以 ANF 为中间表示

- 如何处理嵌套函数、高阶函数？

```
let f x =
  let g y = x + y in g end
in
  (f 2) 3
end
```

→

```
let f x =
  let g y = x + y in g end
in
  let t1 = f 2 in t1 3 end
end
```

```
let f x =
  let g y = x + y in g end
in
  let h t = t 3 in
    h (f 2)
  end
end
```

→

```
let f x =
  let g y = x + y in g end
in
  let h t = t 3 in
    let t1 = f 2 in h t1 end
  end
end
```

转换为显式构造闭包的形式

- 希望转换后所有的函数都是「最外层」函数
- 需要显式进行闭包的构造

「最外层」函数用 @ 标记

函数所需的闭包参数

显式构造闭包

```
let f x =
  let g y = x + y in g end
in
  (f 2) 3
end
```

```
let f x =
  let g y = x + y in g end
in
  let t1 = f 2 in t1 3 end
end
```

```
fun @f {} x =
  let g = clos @g {x} in g end
fun @g {x} y = x + y

let f = clos @f {} in
let t1 = f 2 in t1 3 end end
```

```
let f x =
  let g y = x + y in g end
in
  let h t = t 3 in
    h (f 2)
  end
end
```

```
let f x =
  let g y = x + y in g end
in
  let h t = t 3 in
    let t1 = f 2 in h t1 end
  end
end
```

```
fun @f {} x =
  let g = clos @g {x} in g end
fun @g {x} y = x + y
fun @h {} t = t 3

let f = clos @f {} in
let h = clos @h {} in
let t1 = f 2 in h t1 end end end
```

闭包转换（closure conversion）

$A ::= \text{const}(n) \mid \text{var}(x)$
$C ::= A \mid \text{op}(A_1, A_2) \mid \text{call}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3)$
$E ::= C \mid \text{let}(x, C_1, E_2) \mid \text{letfun}(f, x, E_1, E_2)$

$A ::= \text{const}(n) \mid \text{var}(x)$
$C ::= A \mid \text{op}(A_1, A_2) \mid \text{call}(A_1, A_2) \mid \text{if}(A_1, E_2, E_3)$
$E ::= C \mid \text{let}(x, C_1, E_2) \mid \text{letclos}(f, @\ell, \{\overrightarrow{A_1, \cdots, A_m}\}, E_2)$
$P ::= E \mid \text{fun}(@\ell, \{\overrightarrow{y_1, \cdots, y_m}\}, x, E), P$

- 针对每个 $\text{letfun}(f, x, E_1, E_2)$，做如下处理：
  - 生成一个新的函数标记 $@\ell$
  - 分析函数体定义中的「自由变量」，以它们为闭包参数 $\overrightarrow{y_1, \cdots, y_m}$
  - 在最外层生成一个新的函数 $\text{fun}(@\ell, \{\overrightarrow{y_1, \cdots, y_m}\}, x, E_1)$
  - 把原来的 letfun 位置转换为 $\text{letclos}(f, @\ell, \{\overrightarrow{y_1, \cdots, y_m}\}, E_2)$
- **可以顺便做的优化：如果知道具体的调用函数并且该函数没有自由变量，就不用创建闭包**

闭包转换（closure conversion）

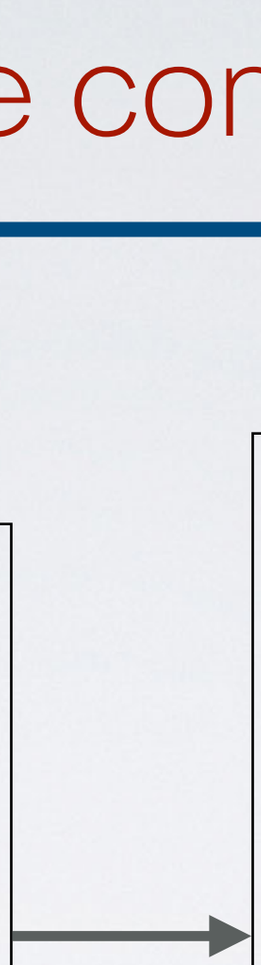
```
let makefac maker =
  let f x = if x == 0 then 1 else x * ((maker maker) (x - 1))
  in f end
in
  let fac = makefac makefac
  in fac 3 end
end
```

↓

```
let makefac maker =
  let f x =
    if x == 0 then 1 else
    let t1 = maker maker in
      let t2 = x - 1 in
        let t3 = t1 t2 in x * t3 end
      end
    end
  in f end
in
  let fac = makefac makefac
  in fac 3 end
end
```

闭包转换（closure conversion）

```
let makefac maker =
  let f x =
    if x == 0 then 1 else
    let t1 = maker maker in
      let t2 = x - 1 in
        let t3 = t1 t2 in x * t3 end
      end
    end
  in f end
in
  let fac = makefac makefac
  in fac 3 end
end
```



```
fun @makefac {} maker =
  let f = clos @f {maker} in f end

fun @f {maker} x =
  if x == 0 then 1 else
  let t1 = maker maker in
    let t2 = x - 1 in
      let t3 = t1 t2 in x * t3 end
    end
  end

let makefac = clos @makefac {} in
  let fac = makefac makefac in
    fac 3
  end
end
```

闭包转换（closure conversion）

```
fun @makefac {} maker =
  let f = clos @f {maker} in f end

fun @f {maker} x =
  if x == 0 then 1 else
  let t1 = maker maker in
    let t2 = x - 1 in
      let t3 = t1 t2 in x * t3 end
    end
  end

let makefac = clos @makefac {} in
  let fac = makefac makefac in
    fac 5
  end
end
```

每个函数都带一个环境参数，记录其使用的自由变量

构造闭包时，记录函数指针 func 和其闭包环境 env

调用函数时，根据闭包的 func 取出函数指针，闭包的 env 取出其求值环境

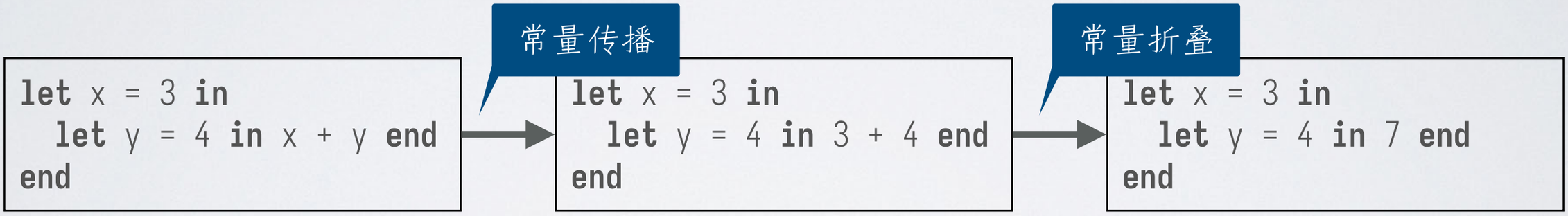
```
function $makefac(env, maker) {
  var f = { func: $f, env: { maker: maker } };
  return f;
}

function $f(env, x) {
  if (x == 0)
    return 1;
  else {
    var t1 = env.maker.func(env.maker.env, env.maker);
    var t2 = x - 1;
    var t3 = t1.func(t1.env, t2);
    return x * t3;
  }
}

function main() {
  var makefac = { func: $makefac, env: {} };
  var fac = makefac.func(makefac.env, makefac);
  return fac.func(fac.env, 5);
}
```

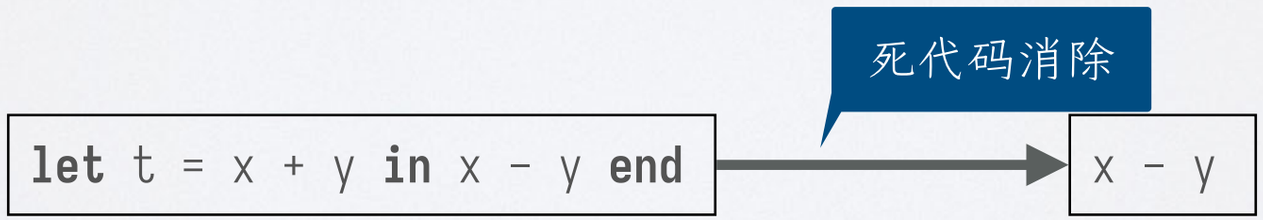
主要内容

- 函数式编程语言 micro-ML
- 中间表示：若干种正规形式
- 前端到中端：源程序到正规形式的转换
- 中端到后端：正规形式程序的闭包转换
- **中端上的优化**

以 ANF 为中间表示

- 回顾：复写传播、常量传播、公共子表达式消除、死代码消除





以 ANF 为中间表示

- 回顾：复写传播、常量传播、公共子表达式消除、死代码消除

```
let y = 4 in
  let z = y in
    let t1 = x + y in
      let t2 = x + z in
        t1 * t2
      end
    end
  end
end
```

→

```
let y = 4 in
  let z = y in
    let t1 = x + y in
      let t2 = x + y in
        t1 * t2
      end
    end
  end
end
```

→

```
let y = 4 in
  let z = 4 in
    let t1 = x + 4 in
      let t2 = x + 4 in
        t1 * t2
      end
    end
  end
end
```

↓

```
let y = 4 in
  let z = 4 in
    let t1 = x + 4 in
      let t2 = t1 in
        t1 * t2
      end
    end
  end
end
```

→

```
let y = 4 in
  let z = y in
    let t1 = x + 4 in
      let t2 = t1 in
        t1 * t2
      end
    end
  end
end
```

- - - →

```
let t1 = x + 4 in
  t1 * t1
end
```

复写传播

- 对于 **let** x = y **in** exp **end**，把 exp 中对 x 的使用替换为 y

产生规则	属性计算规则
$A \to \mathsf{const}(n)$	$A.\mathsf{anf} = \mathsf{const}(n)$
$A \to \mathsf{var}(x)$	$A.\mathsf{anf} = \mathsf{var}(A.\sigma(x))$
$C \to \mathsf{op}(A_1, A_2)$	$A_1.\sigma = A_2.\sigma = C.\sigma$ $\quad C.\mathsf{anf} = \mathsf{op}(A_1.\mathsf{anf}, A_2.\mathsf{anf})$
$C \to \mathsf{call}(A_1, A_2)$	$A_1.\sigma = A_2.\sigma = C.\sigma$ $\quad C.\mathsf{anf} = \mathsf{call}(A_1.\mathsf{anf}, A_2.\mathsf{anf})$
$C \to \mathsf{if}(A_1, E_2, E_3)$	$A_1.\sigma = E_2.\sigma = E_3.\sigma = C.\sigma$ $\quad C.\mathsf{anf} = \mathsf{if}(A_1.\mathsf{anf}, E_2.\mathsf{anf}, E_3.\mathsf{anf})$
$E \to \mathsf{let}(x, C_1, E_2)$	$C_1.\sigma = E.\sigma$ $\quad E_2.\sigma = [x \mapsto (C_1.\mathsf{anf}\ \mathsf{is}\ \mathsf{var}(y)\ ?\ y : x)]E.\sigma$ $E.\mathsf{anf} = \mathsf{let}(x, C_1.\mathsf{anf}, E_2.\mathsf{anf})$
$E \to \mathsf{letfun}(f, x, E_1, E_2)$	$E_1.\sigma = [x \mapsto x, f \mapsto f]E.\sigma$ $\quad E_2.\sigma = [f \mapsto f]E.\sigma$ $E.\mathsf{anf} = \mathsf{letfun}(f, x, E_1.\mathsf{anf}, E_2.\mathsf{anf})$

继承属性 σ 记录变量名到变量名的映射

如果形如 let x = y，则在 σ 中记录该复写

常量传播

- 对于 **let** x = *INT* **in** exp **end**，把 exp 中对 x 的使用替换为 *INT*
- 对于 *INT* op *INT*，替换为运算结果(即常量折叠)

产生规则	属性计算规则
$A \to \mathsf{const}(n)$	$A.\mathsf{anf} = \mathsf{const}(n)$
$A \to \mathsf{var}(x)$	$A.\mathsf{anf} = x \in A.\sigma \ ?\ \mathsf{const}(A.\sigma(x)) : \mathsf{var}(x)$
$C \to \mathsf{op}(A_1, A_2)$	$A_1.\sigma = A_2.\sigma = C.\sigma \quad C.\mathsf{anf} = A_1.\mathsf{anf}\ \mathsf{is}\ \mathsf{const}(n_1) \wedge A_2.\mathsf{anf}\ \mathsf{is}\ \mathsf{const}(n_2)\ ?\ \mathsf{const}(f_{\mathsf{op}}(n_1, n_2)) : \mathsf{op}(A_1.\mathsf{anf}, A_2.\mathsf{anf})$
$C \to \mathsf{call}(A_1, A_2)$	$A_1.\sigma = A_2.\sigma = C.\sigma \quad C.\mathsf{anf} = \mathsf{call}(A_1.\mathsf{anf}, A_2.\mathsf{anf})$
$C \to \mathsf{if}(A_1, E_2, E_3)$	$A_1.\sigma = E_2.\sigma = E_3.\sigma = C.\sigma \quad C.\mathsf{anf} = \mathsf{if}(A_1.\mathsf{anf}, E_2.\mathsf{anf}, E_3.\mathsf{anf})$
$E \to \mathsf{let}(x, C_1, E_2)$	$C_1.\sigma = E.\sigma \quad E_2.\sigma = C_1.\mathsf{anf}\ \mathsf{is}\ \mathsf{const}(n)\ ?\ [x \mapsto n]E.\sigma : E.\sigma \backslash \{x\}$ $E.\mathsf{anf} = \mathsf{let}(x, C_1.\mathsf{anf}, E_2.\mathsf{anf})$
$E \to \mathsf{letfun}(f, x, E_1, E_2)$	$E_1.\sigma = E.\sigma \backslash \{f, x\} \quad E_2.\sigma = E.\sigma \backslash \{f\}$ $E.\mathsf{anf} = \mathsf{letfun}(f, x, E_1.\mathsf{anf}, E_2.\mathsf{anf})$

继承属性 σ 记录变量名到常量的映射

如果运算分量都是常量则折叠

不是常量的变量要移除

公共子表达式消除

- 对于 **let** x = a op b **in** exp **end**，把 exp 中相同的 a op b 替换为 x，需要注意 a 和 b 是变量时要考虑变量覆盖问题

继承属性 σ 记录表达式到变量的映射

产生规则	属性计算规则
$C \to \mathsf{op}(A_1, A_2)$	$C.\mathsf{anf} = \mathsf{op}(A_1, A_2) \in C.\sigma \ ? \ C.\sigma(\mathsf{op}(A_1, A_2)) : \mathsf{op}(A_1, A_2)$
$C \to \mathsf{call}(A_1, A_2)$	$C.\mathsf{anf} = \mathsf{call}(A_1, A_2)$
$C \to \mathsf{if}(A_1, E_2, E_3)$	$E_2.\sigma = E_3.\sigma = C.\sigma \qquad C.\mathsf{anf} = \mathsf{if}(A_1, E_2.\mathsf{anf}, E_3.\mathsf{anf})$
$E \to \mathsf{let}(x, C_1, E_2)$	$C_1.\sigma = E.\sigma$ $E_2.\sigma = C_1.\mathsf{anf}\ \mathsf{is}\ \mathsf{op}(a,b) \ ? \ [\mathsf{op}(a,b) \mapsto x](E.\sigma - x) : (E.\sigma - x)$ $E.\mathsf{anf} = \mathsf{let}(x, C_1.\mathsf{anf}, E_2.\mathsf{anf})$
$E \to \mathsf{letfun}(f, x, E_1, E_2)$	$E_1.\sigma = E.\sigma - f - x \qquad E_2.\sigma = E.\sigma - f$ $E.\mathsf{anf} = \mathsf{letfun}(f, x, E_1.\mathsf{anf}, E_2.\mathsf{anf})$

去掉之前记录的涉及 x 的表达式

为了简化变量名的处理，可以在优化前改写一遍所有变量名，使得它们都不相同

死代码消除

- 对于 **let** x = ... **in** exp **end**，若 exp 中没有使用 x，则可以把这个 **let** 绑定消除

综合属性 fv 记录出现的自由变量的集合

产生规则	属性计算规则	
$A \to \mathsf{const}(n)$	$A.\mathsf{fv} = \varnothing$	
$A \to \mathsf{var}(x)$	$A.\mathsf{fv} = \{x\}$	
$C \to \mathsf{op}(A_1, A_2)$	$C.\mathsf{fv} = A_1.\mathsf{fv} \cup A_2.\mathsf{fv}$	$C.\mathsf{anf} = \mathsf{op}(A_1, A_2)$
$C \to \mathsf{call}(A_1, A_2)$	$C.\mathsf{fv} = A_1.\mathsf{fv} \cup A_2.\mathsf{fv}$	$C.\mathsf{anf} = \mathsf{call}(A_1, A_2)$
$C \to \mathsf{if}(A_1, E_2, E_3)$	$C.\mathsf{fv} = A_1.\mathsf{fv} \cup E_2.\mathsf{fv} \cup E_3.\mathsf{fv}$	$C.\mathsf{anf} = \mathsf{if}(A_1, E_2.\mathsf{anf}, E_3.\mathsf{anf})$
$E \to \mathsf{let}(x, C_1, E_2)$	$E.\mathsf{anf} = x \in E_2.\mathsf{fv}\ ?\ \mathsf{let}(x, C_1.\mathsf{anf}, E_2.\mathsf{anf}) : E_2.\mathsf{anf}$ $E.\mathsf{fv} = x \in E_2.\mathsf{fv}\ ?\ (C_1.\mathsf{fv} \cup (E_2.\mathsf{fv} \setminus \{x\})) : E_2.\mathsf{fv}$	
$E \to \mathsf{letfun}(f, x, E_1, E_2)$	$E.\mathsf{anf} = f \in E_2.\mathsf{fv}\ ?\ \mathsf{letfun}(f, x, E_1.\mathsf{anf}, E_2.\mathsf{anf}) : E_2.\mathsf{anf}$ $E.\mathsf{fv} = f \in E_2.\mathsf{fv}\ ?\ ((E_1.\mathsf{fv} \setminus \{f, x\}) \cup (E_2.\mathsf{fv} \setminus \{f\})) : E_2.\mathsf{fv}$	

如果 x 不出现在 E_2 使用的自由变量里则消除

E_1 是函数体定义，其内部使用的函数名 f 和参数名 x 不是自由变量

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