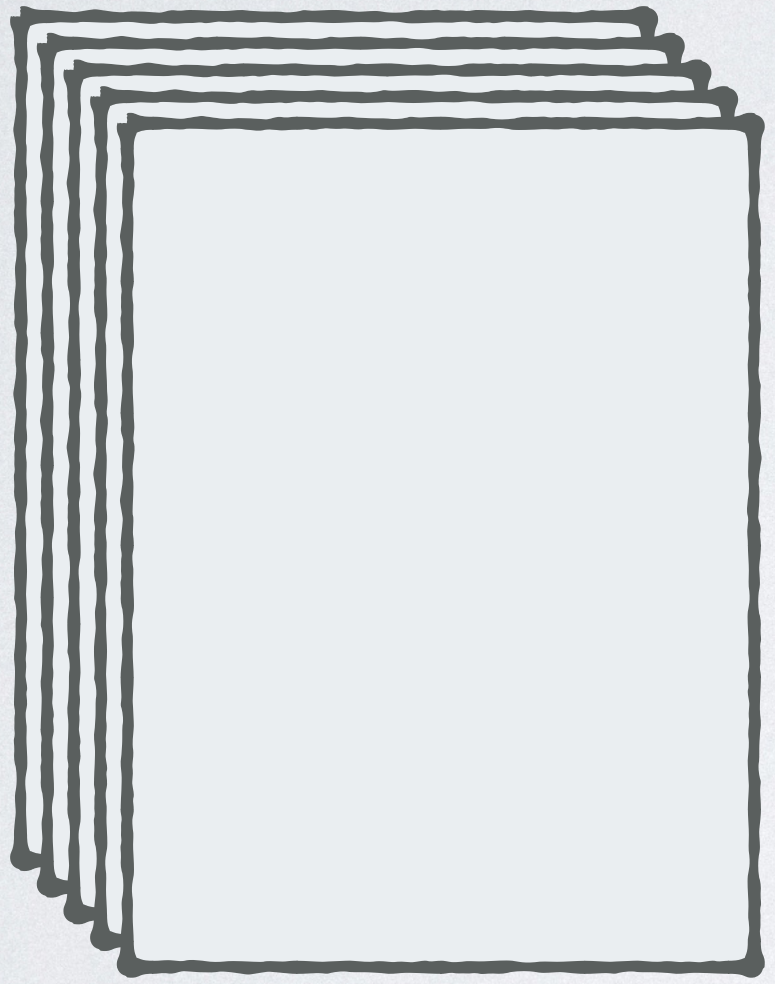
Type-Guided Worst-Case Input Generation

Di Wang, Jan Hoffmann

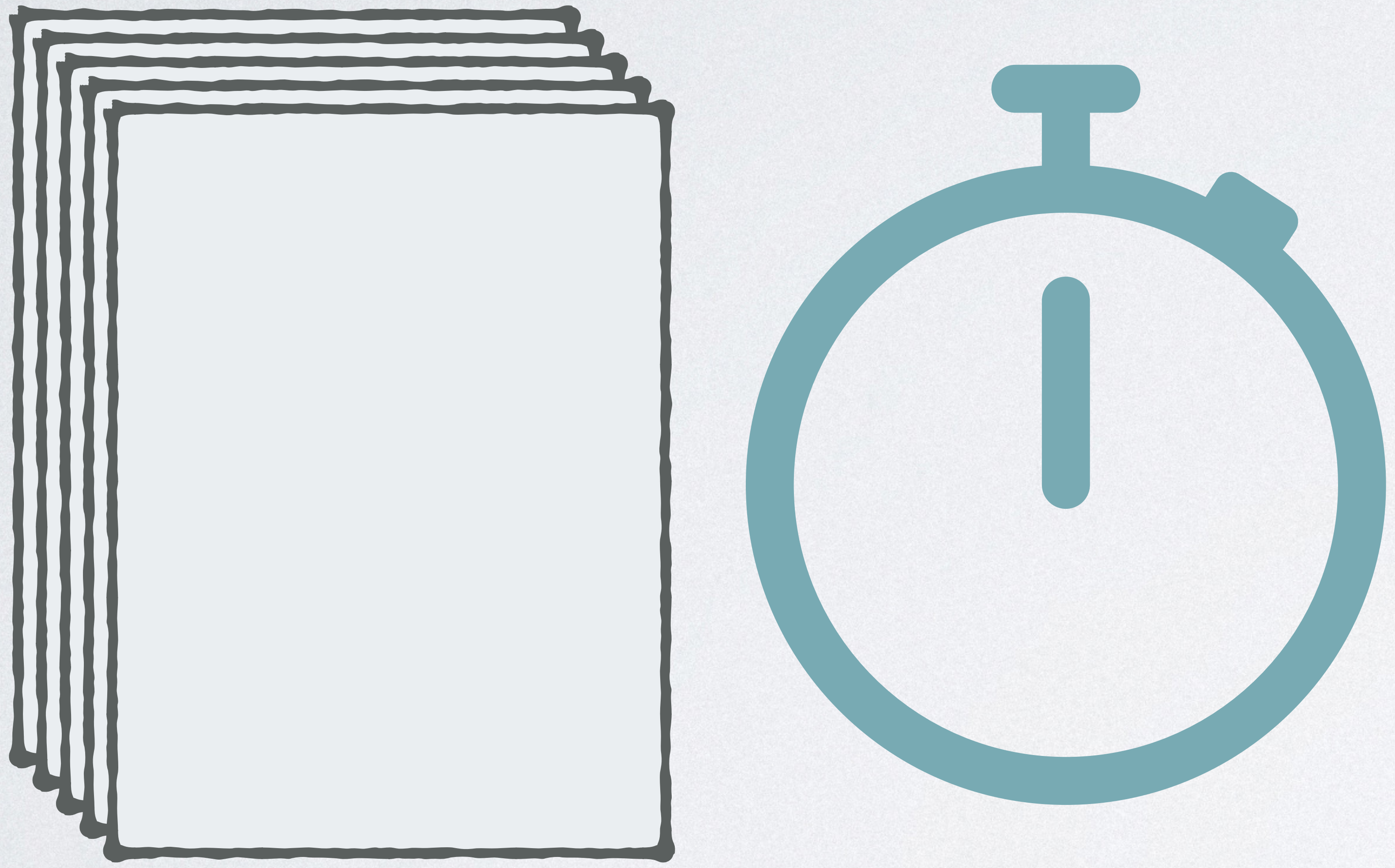
Carnegie Mellon University

Resource Analysis



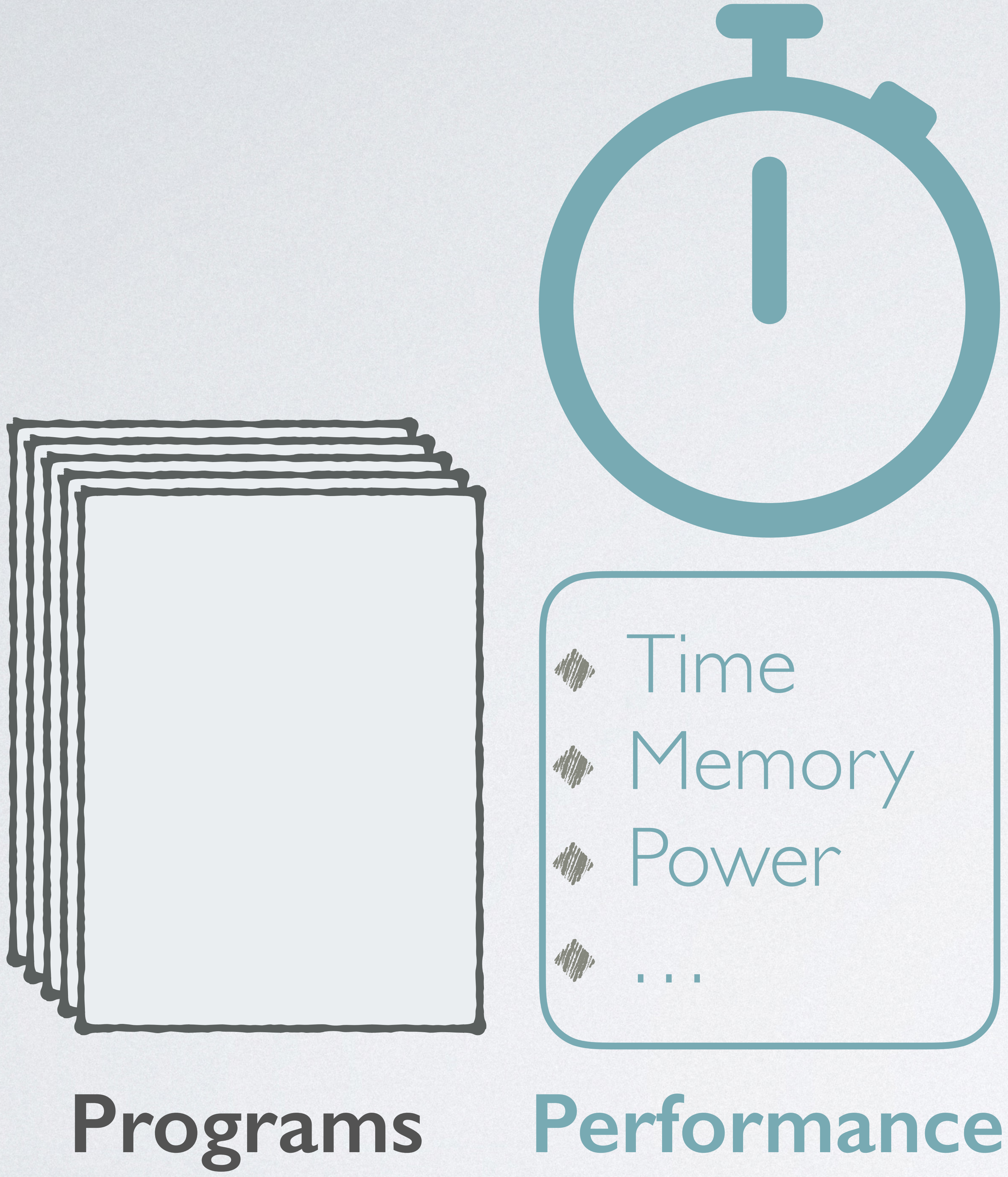
Programs

Resource Analysis



Programs **Performance**

Resource Analysis



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- Memory
- Power
- …

Programs **Performance**

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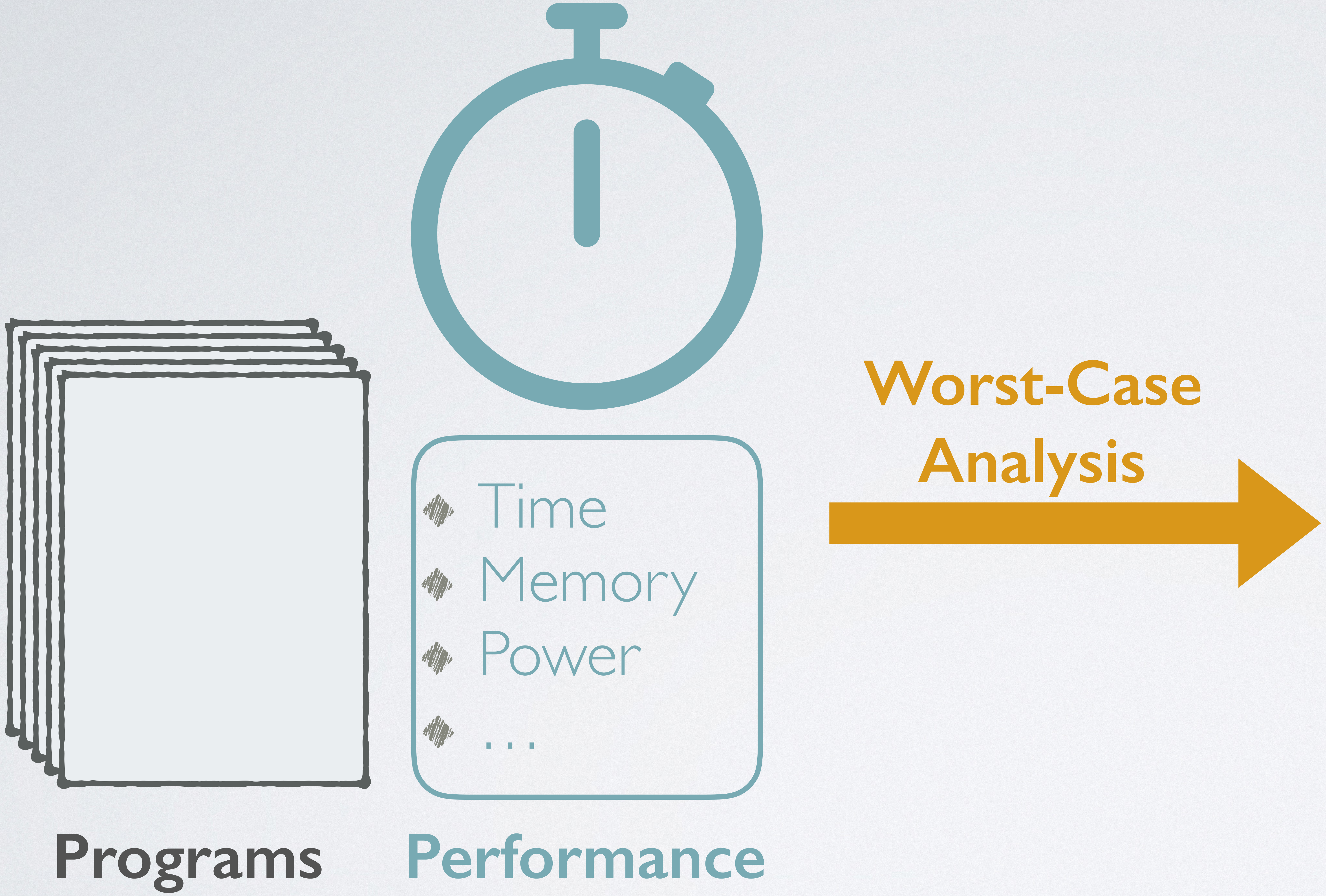

- Time
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- …

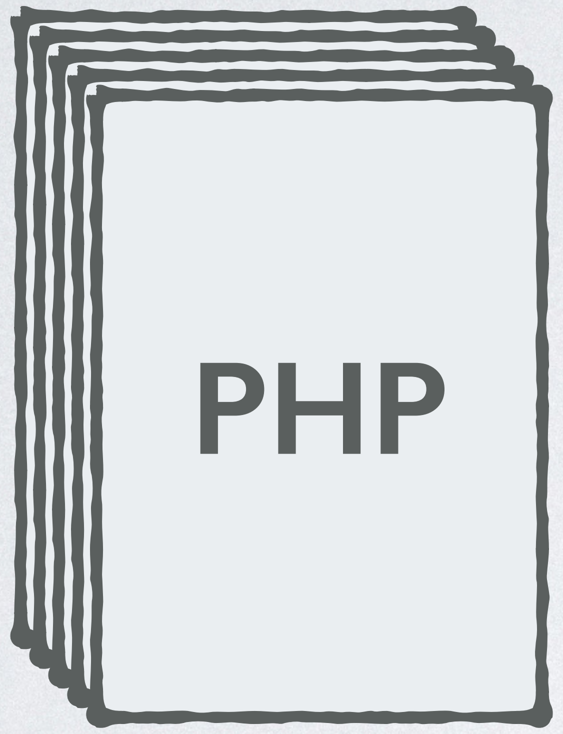
Programs

Performance

Worst-Case Analysis

- Performance bottlenecks
- Algorithmic complexity vulnerabilities
- Timing side channels

Example of Worst-Case Analysis



[1] CVE - CVE-2011-4885. Available on: https://cve.mitre.org/cgi-bin/cvename.cgi?name=CVE-2011-4885.

[2] PHP 5.3.8 - Hashtables Denial of Service. Available on https://www.exploit-db.com/exploits/18296/.

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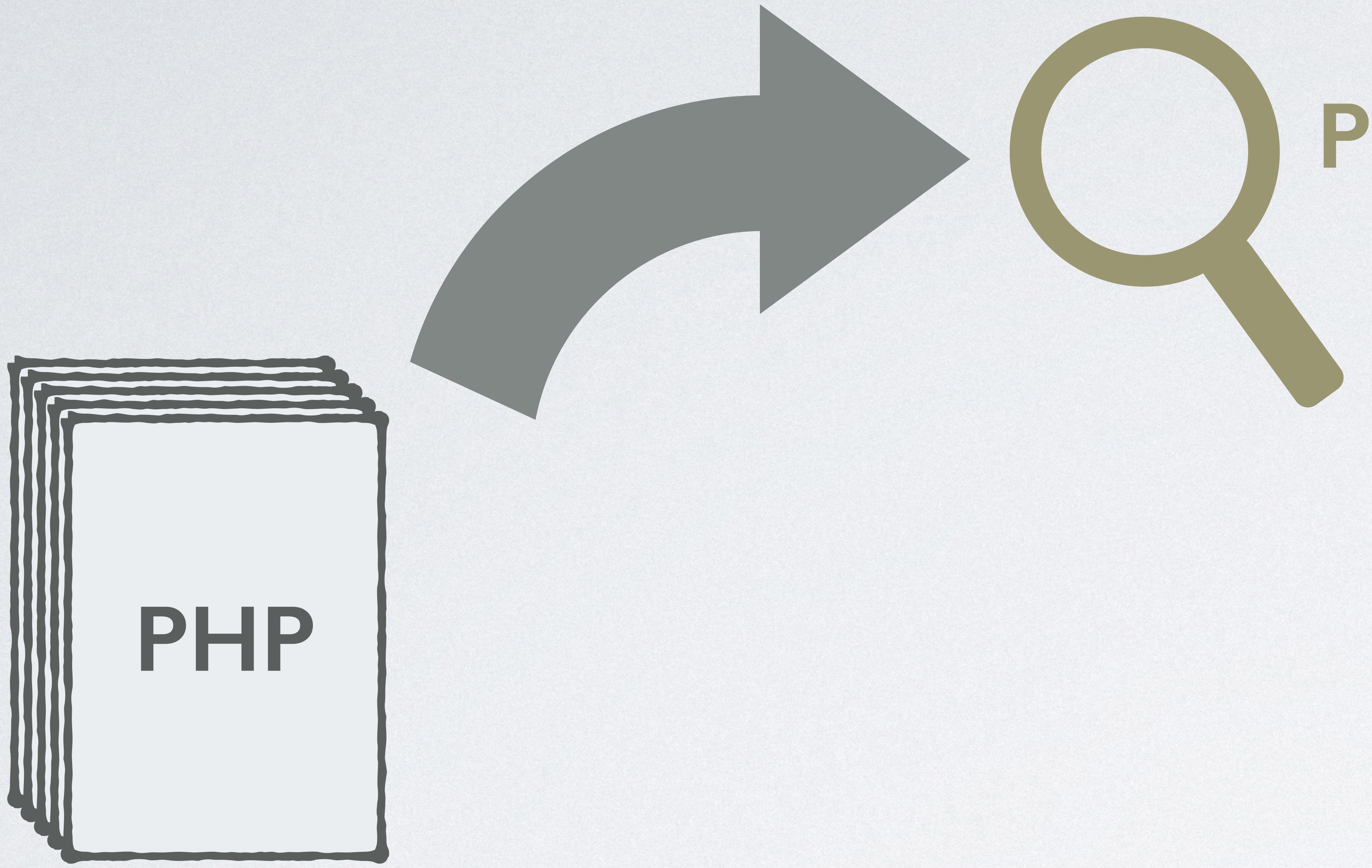

Potential Denial-of-Service attack[1]

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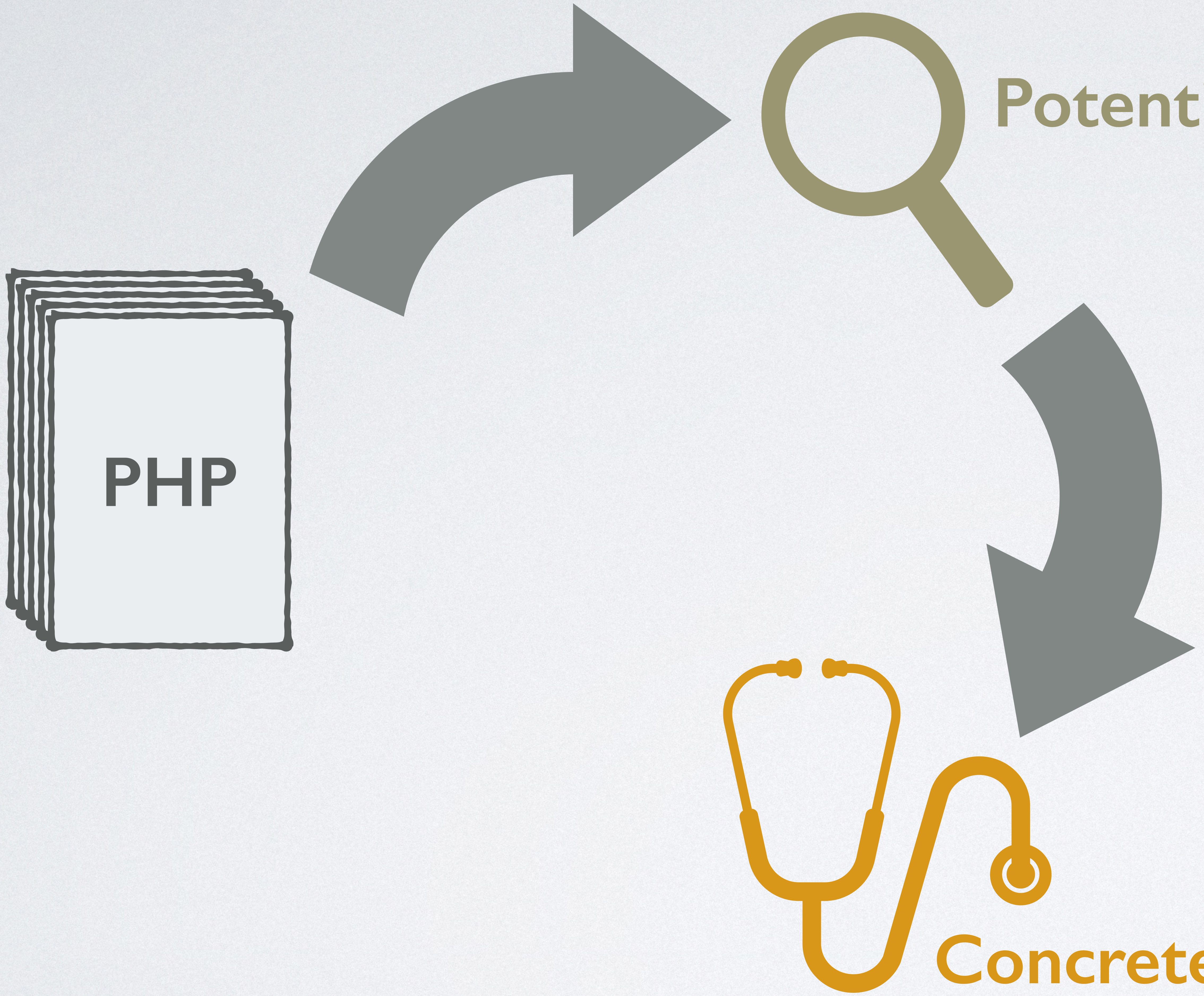

PHP

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Concrete exploits (by hash collisions)[2]

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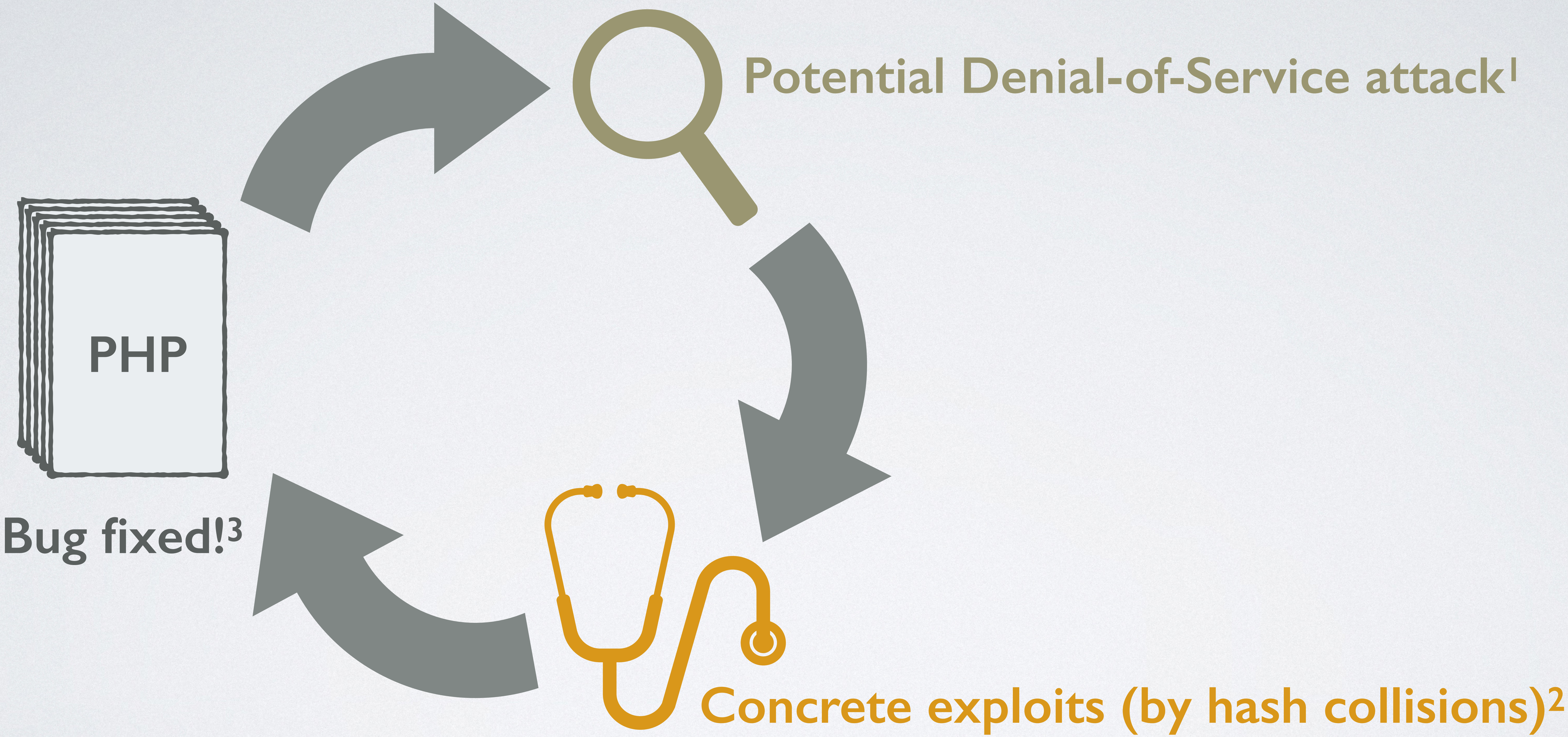

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Bug fixed![3]

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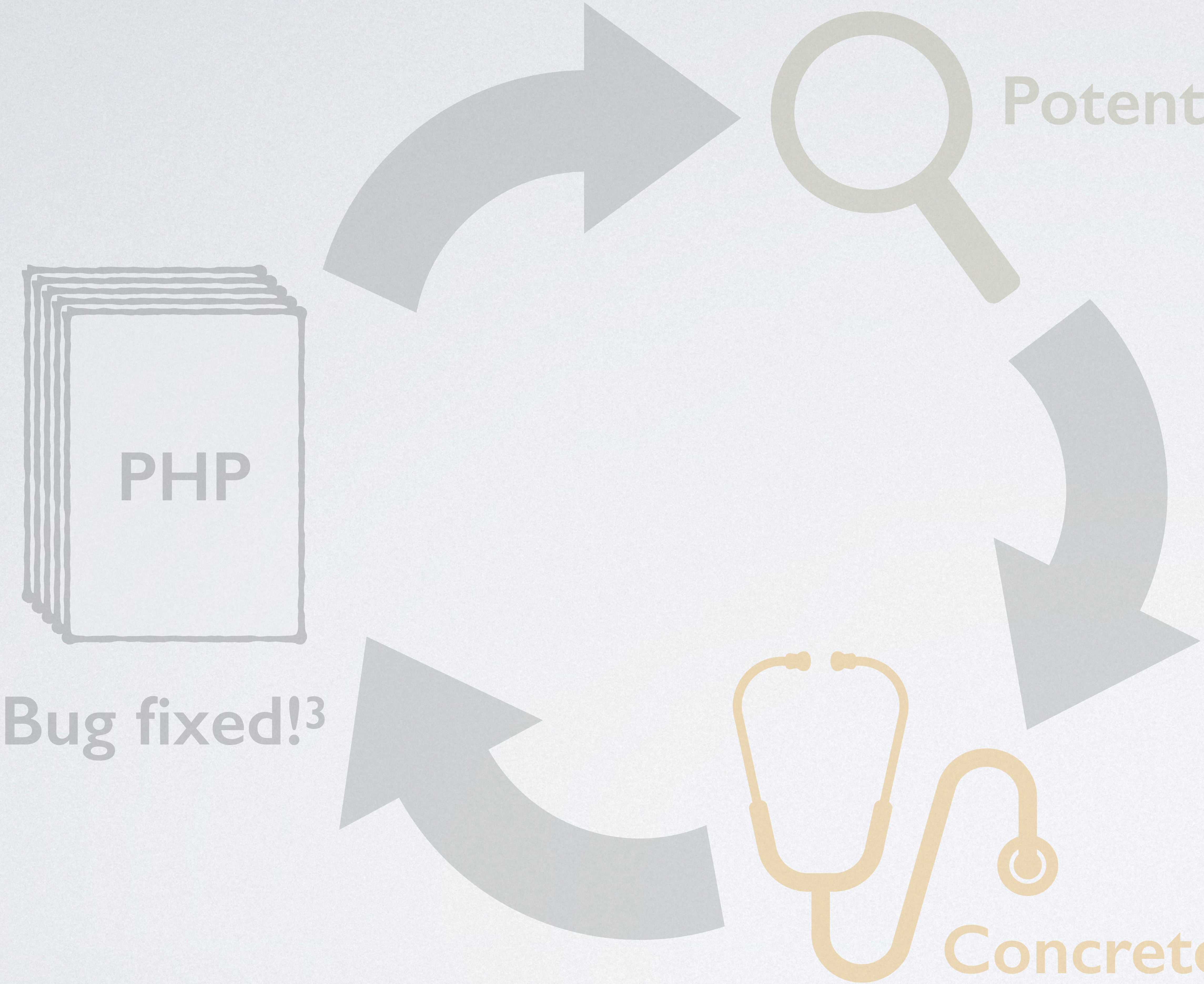

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PHP

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Worst-case inputs are instrumental to understand and fix performance bugs!

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Existing Approaches

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Dynamic

- Fuzz testing
- Symbolic execution
- Dynamic worst-case analysis
- …

- Flexible & universal
- Potentially unsound: The resulting inputs might not expose the worst-case behavior.

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Static

- Type systems
- Abstract interpretation
- …

- Sound upper bounds
- Potentially not tight: No concrete witness — the bound might be too conservative.

Contributions

- A **type-guided** **worst-case input** generation algorithm
- Proof of soundness and relative completeness
- Heuristics to improve scalability

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Resource Aware ML (RaML)

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Symbolic Execution

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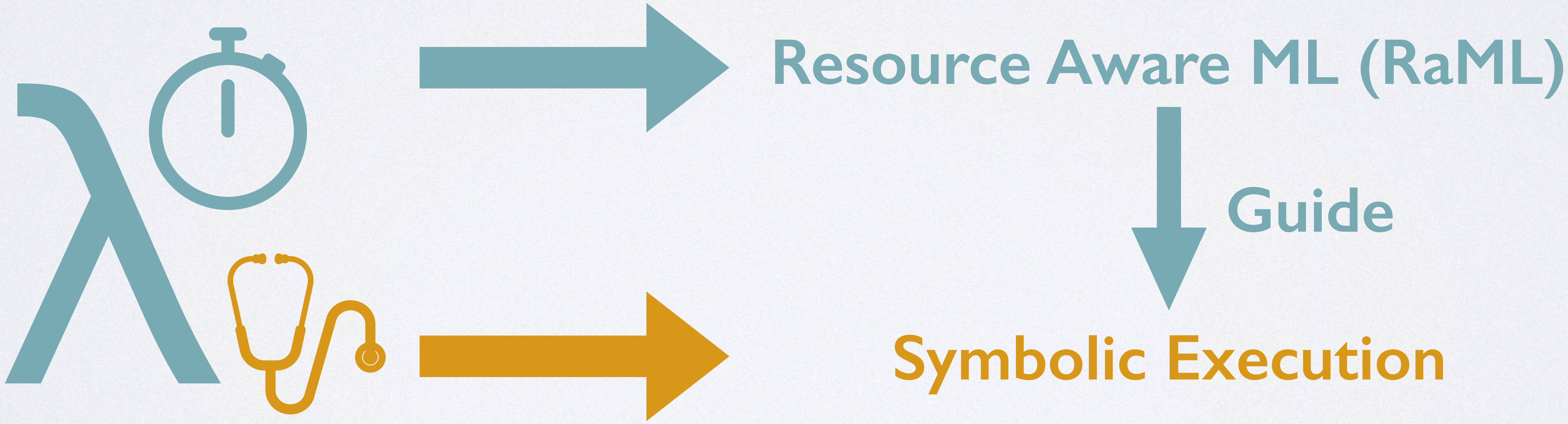

Resource Aware ML (RaML)

Guide

Symbolic Execution

Overview

- ☑ Motivation
- □ Resource Aware ML (RaML)
- □ Type-Guided Worst-Case Input Generation
- □ Evaluation

Amortized Resource Analysis

- The potential method

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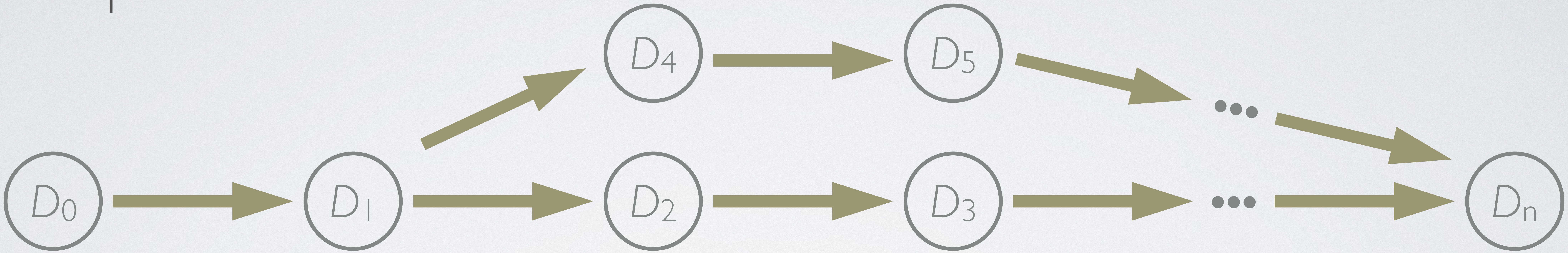

Amortized Resource Analysis

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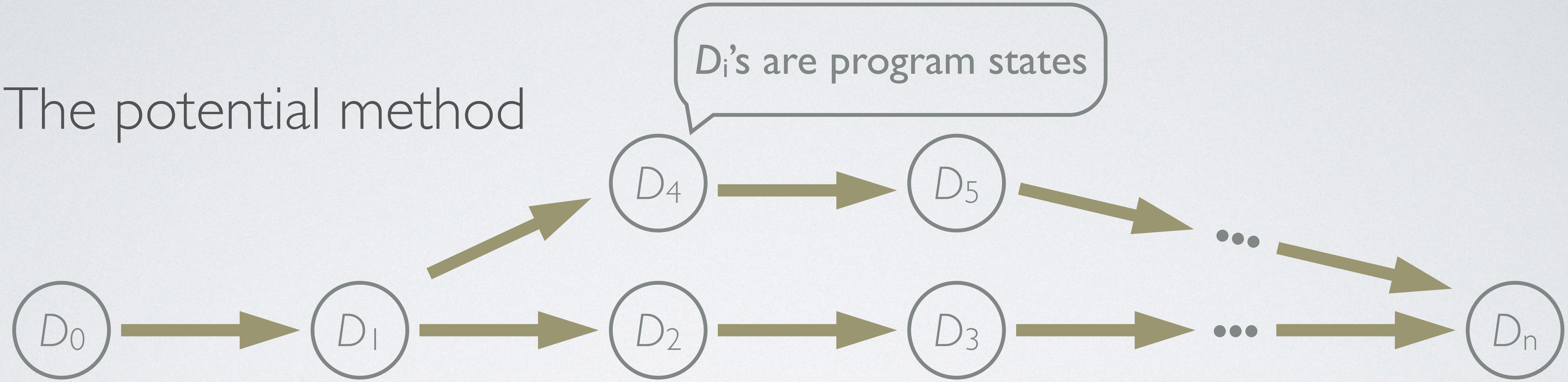

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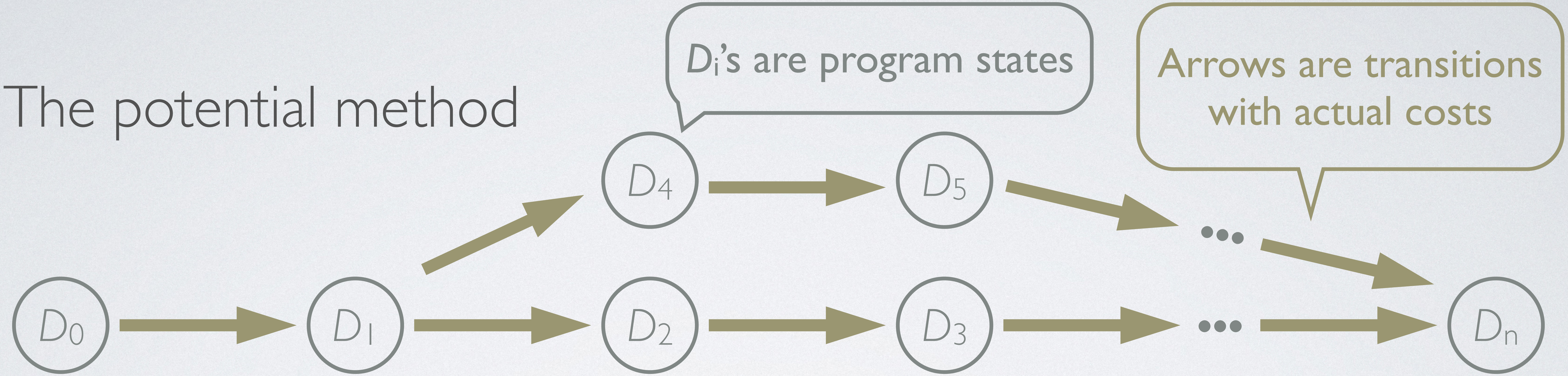
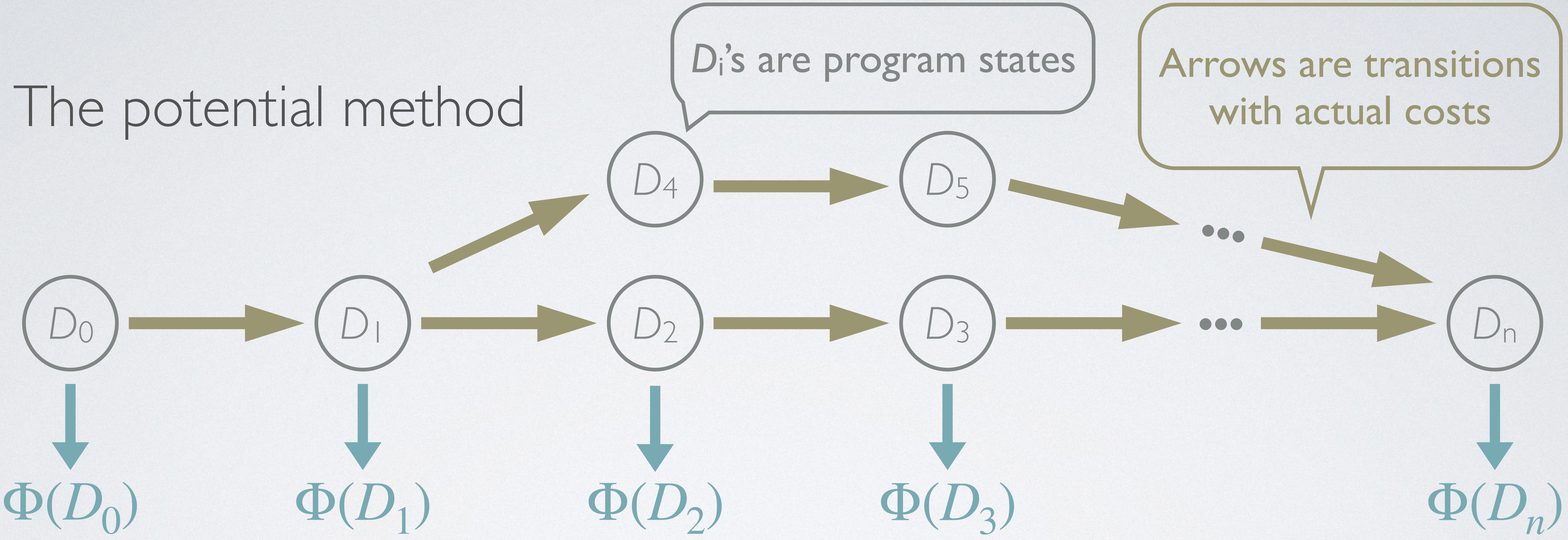


D_i's are program states

Arrows are transitions with actual costs

AMORTIZED RESOURCE ANALYSIS

- The potential method



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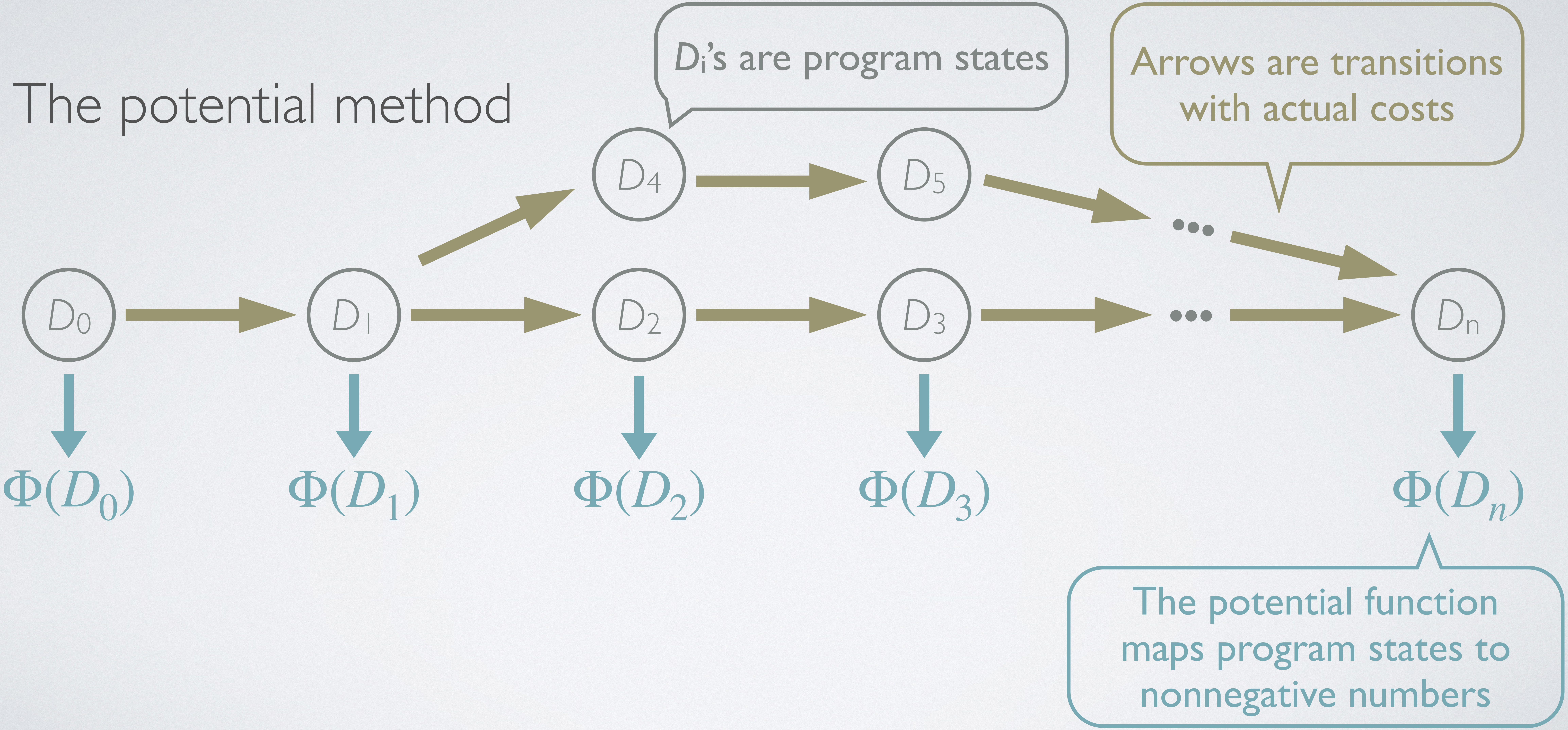
D_i's are program states
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D_0
D_1
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D_4
D_5
...
...
D_n
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The potential function maps program states to nonnegative numbers

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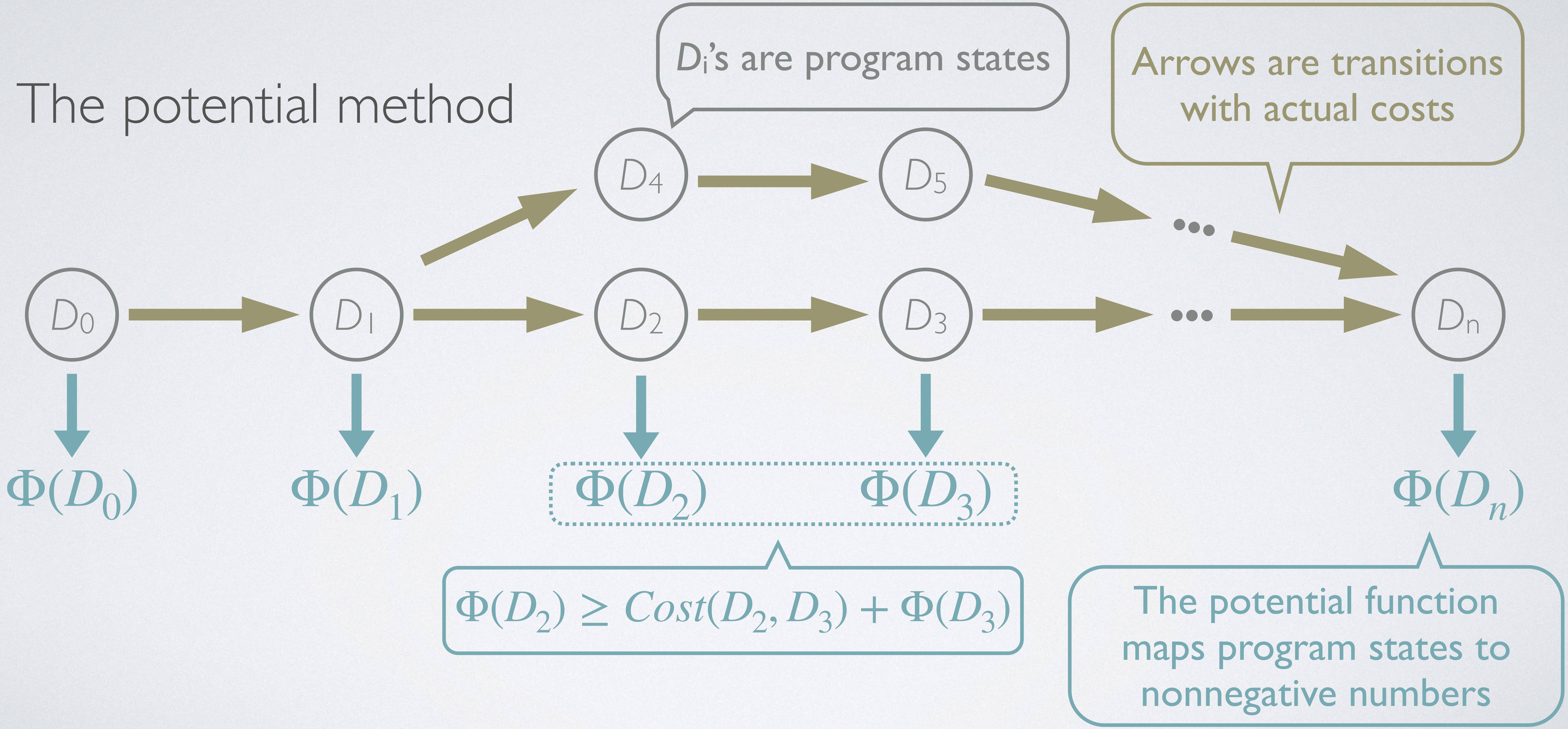

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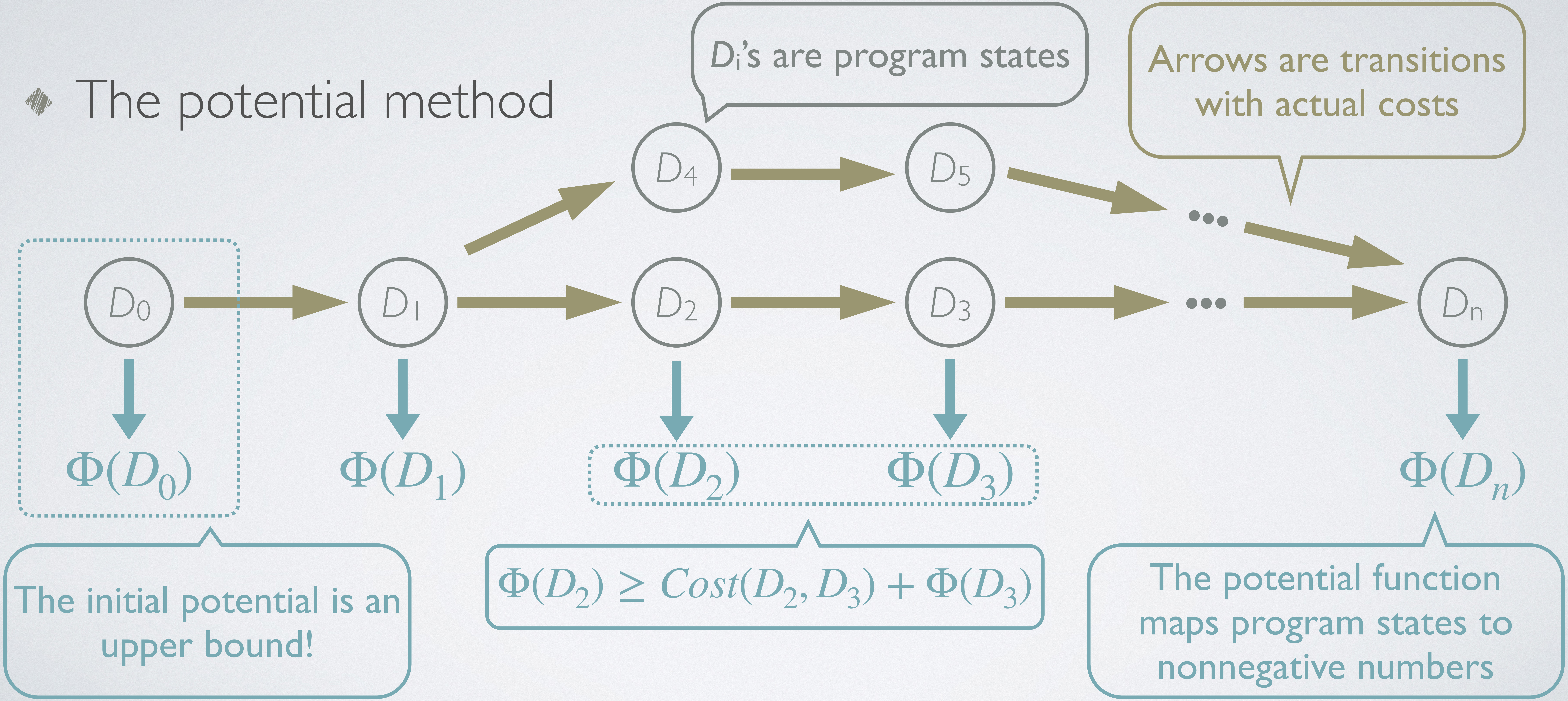

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The initial potential is an upper bound!
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Type-Based Analysis

```
let rec lpairs l =
  match l with
  | [] -> []
  | x1 :: xs ->
    match xs with
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      if (x1:int) < (x2:int) then
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- The **potential** at a program point is defined by a **static** annotation of data structures.

- A list of length ***n*** annotated with a nonnegative number ***q*** has ***q·n*** units of potential.

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Each of [ ], : :, (,) consumes 2 memory cells.

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$Cost = 2 \cdot |\ell| + 2$

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$\Phi_1 = 2 \cdot |xs| + 4$

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$$\Phi_2 = 2 \cdot |xs'| + 6$$

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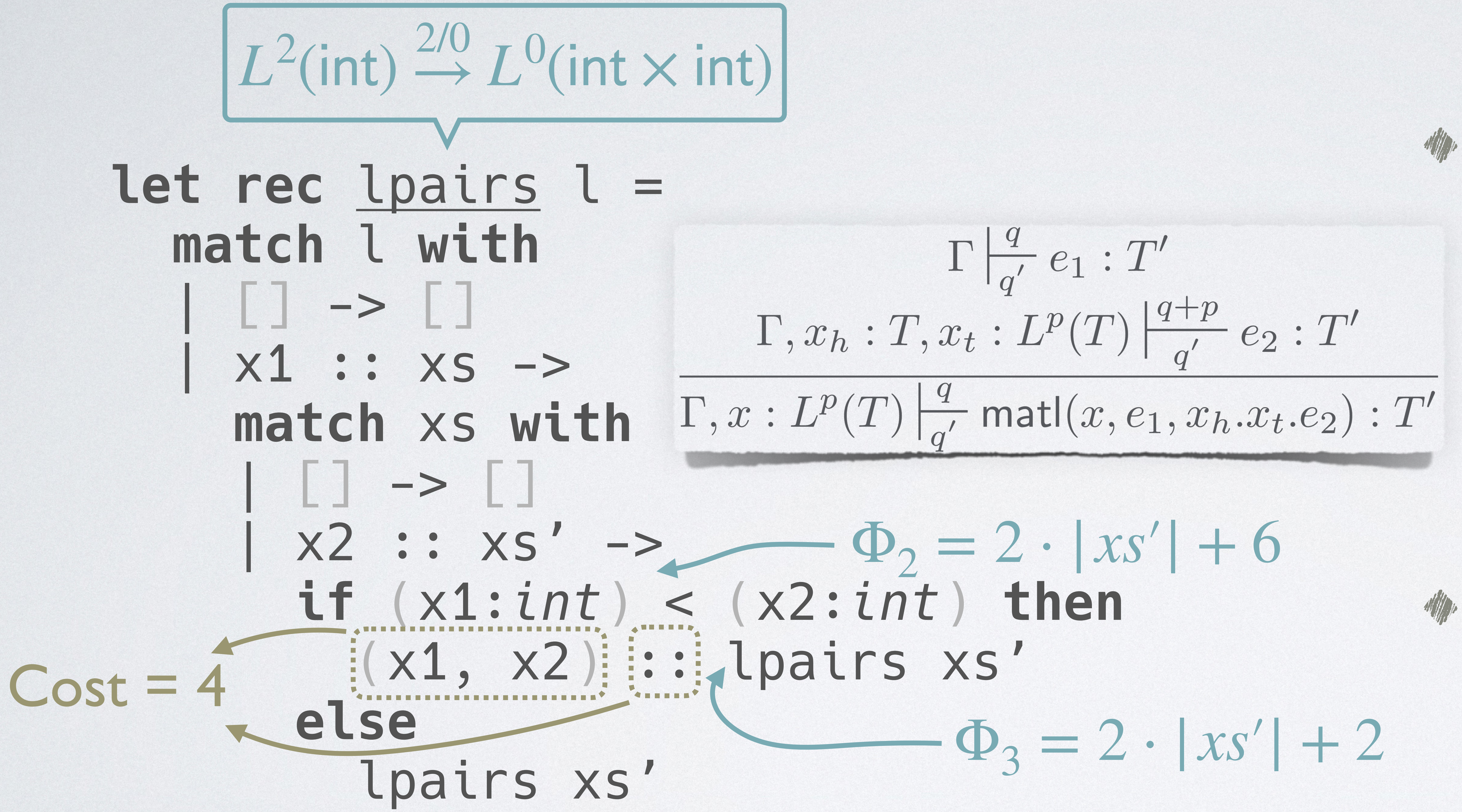

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$$\frac{\begin{array}{c}\Gamma \overset{q}{\underset{q'}{\vdash}} e_1 : T' \\ \Gamma, x_h : T, x_t : L^p(T) \overset{q+p}{\underset{q'}{\vdash}} e_2 : T'\end{array}}{\Gamma, x : L^p(T) \overset{q}{\underset{q'}{\vdash}} \mathsf{matl}(x, e_1, x_h.x_t.e_2) : T'}$$

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OVERVIEW

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- ☑ Resource Aware ML (RaML)
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- □ Evaluation

Symbolic Execution

- **Idea**: **search** all execution paths, **record** path constraints, and **compute** resource usage

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symbolic environment

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$$\underset{\text{symbolic environment}}{\gamma} \vdash \underset{\substack{\downarrow \\ \text{expression}}}{e} \Rightarrow \langle \psi, S \rangle$$

Symbolic Execution

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expression

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symbolic evaluation result

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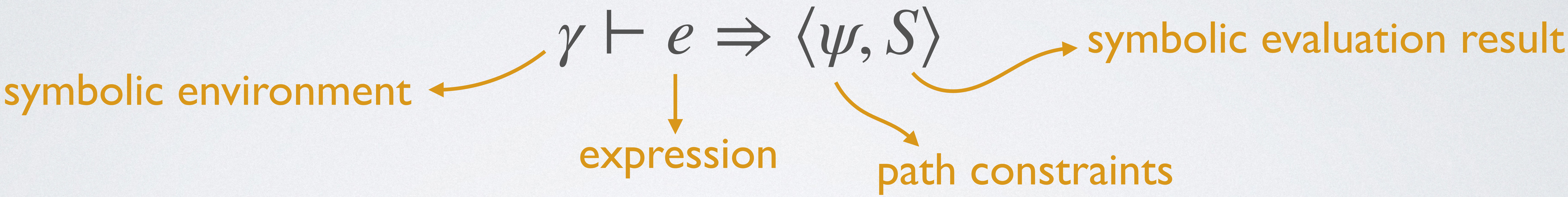

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- Symbolic execution rules for conditional expressions

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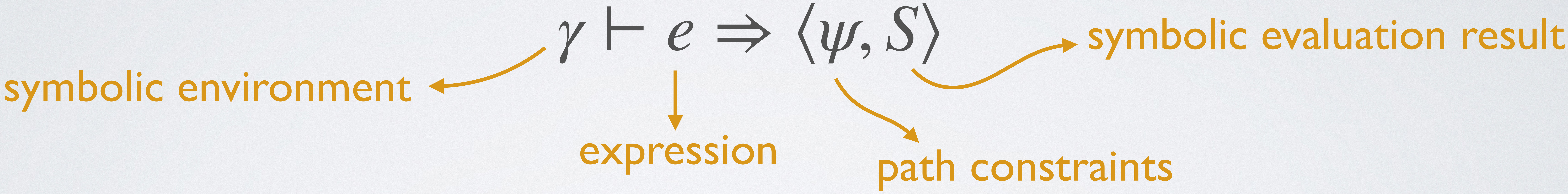

$$\gamma \vdash e \Rightarrow \langle \psi, S \rangle$$

- Symbolic execution rules for conditional expressions

$$\boxed{\text{Then}} \quad \frac{\gamma \vdash e_1 \Rightarrow \langle \psi, S \rangle}{\gamma \vdash \text{if } e \text{ then } e_1 \text{ else } e_2 \Rightarrow \langle \gamma(e) \wedge \psi, S \rangle}$$

$$\boxed{\text{Else}} \quad \frac{\gamma \vdash e_2 \Rightarrow \langle \psi, S \rangle}{\gamma \vdash \text{if } e \text{ then } e_1 \text{ else } e_2 \Rightarrow \langle \neg\gamma(e) \wedge \psi, S \rangle}$$

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- An example of worst-case execution paths for input lists of length 4

$$\begin{aligned}&\ell \mapsto [\mathrm{int}^1, \mathrm{int}^2, \mathrm{int}^3, \mathrm{int}^4] \vdash \\ &\quad \mathrm{lpairs}\ \ell \Rightarrow \langle (\mathrm{int}^1 < \mathrm{int}^2) \wedge (\mathrm{int}^3 < \mathrm{int}^4), \\ &\qquad\qquad [(\mathrm{int}^1, \mathrm{int}^2), (\mathrm{int}^3, \mathrm{int}^4)] \rangle\end{aligned}$$

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$$\mathrm{lpairs}\ \ell \Rightarrow \langle (\mathrm{int}^1 < \mathrm{int}^2) \wedge (\mathrm{int}^3 < \mathrm{int}^4), [(\mathrm{int}^1, \mathrm{int}^2), (\mathrm{int}^3, \mathrm{int}^4)] \rangle$$

- Invoke an SMT solver to find a model, e.g., `[0,1,0,1]`

Type-Guided Symbolic Execution

- **Nondeterminism** leads to state explosion

Then

$$\frac{\gamma \vdash e_1 \Rightarrow \langle \psi, S \rangle}{\gamma \vdash \textsf{if } e \textsf{ then } e_1 \textsf{ else } e_2 \Rightarrow \langle \gamma(e) \wedge \psi, S \rangle}$$

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Use the information about **potentials** obtained from **resource aware type checking** to **prune the search space** of symbolic execution.

Type-Guided Symbolic Execution

$L^2(\text{int}) \xrightarrow{2/0} L^0(\text{int} \times \text{int})$

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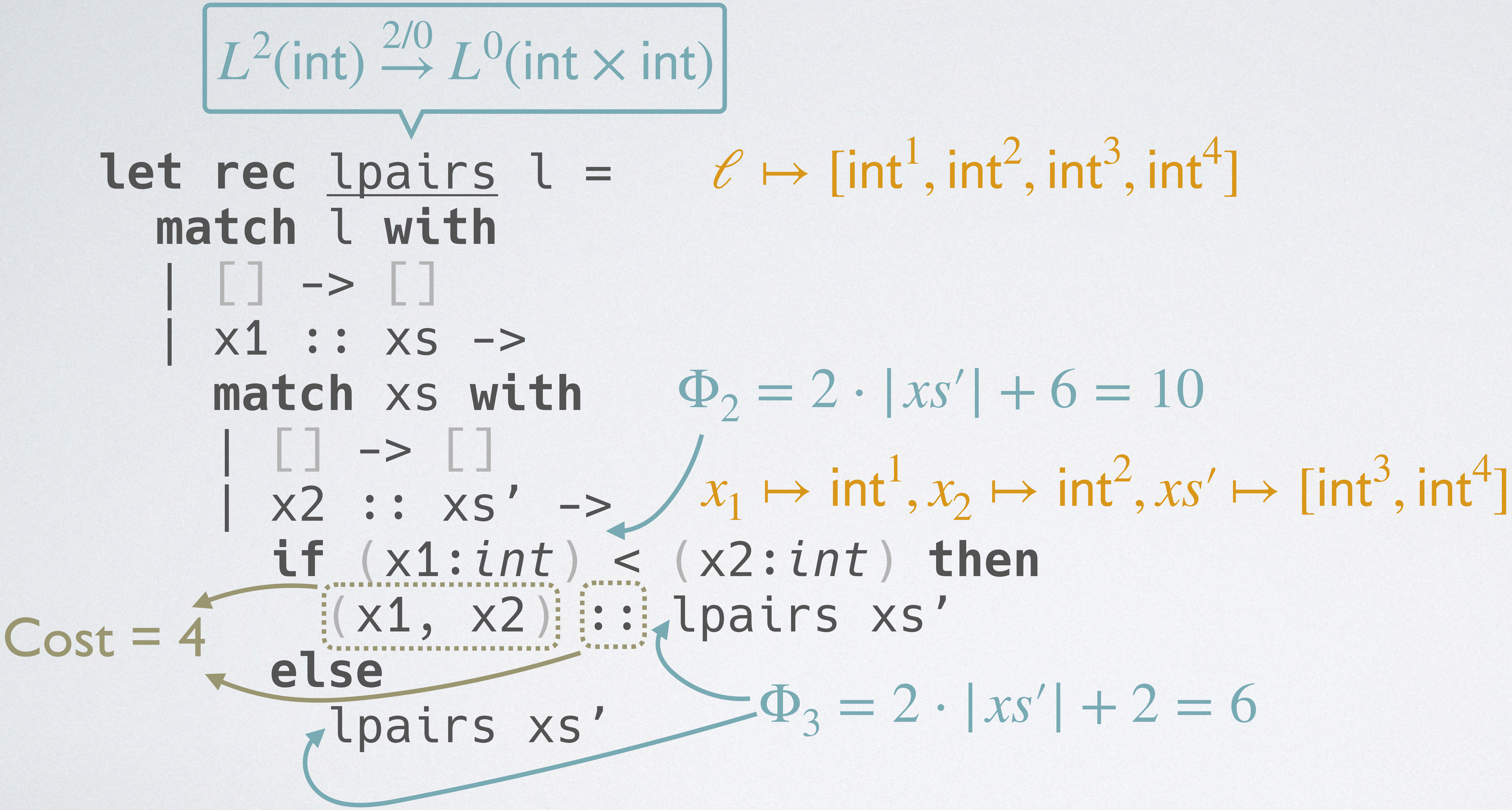

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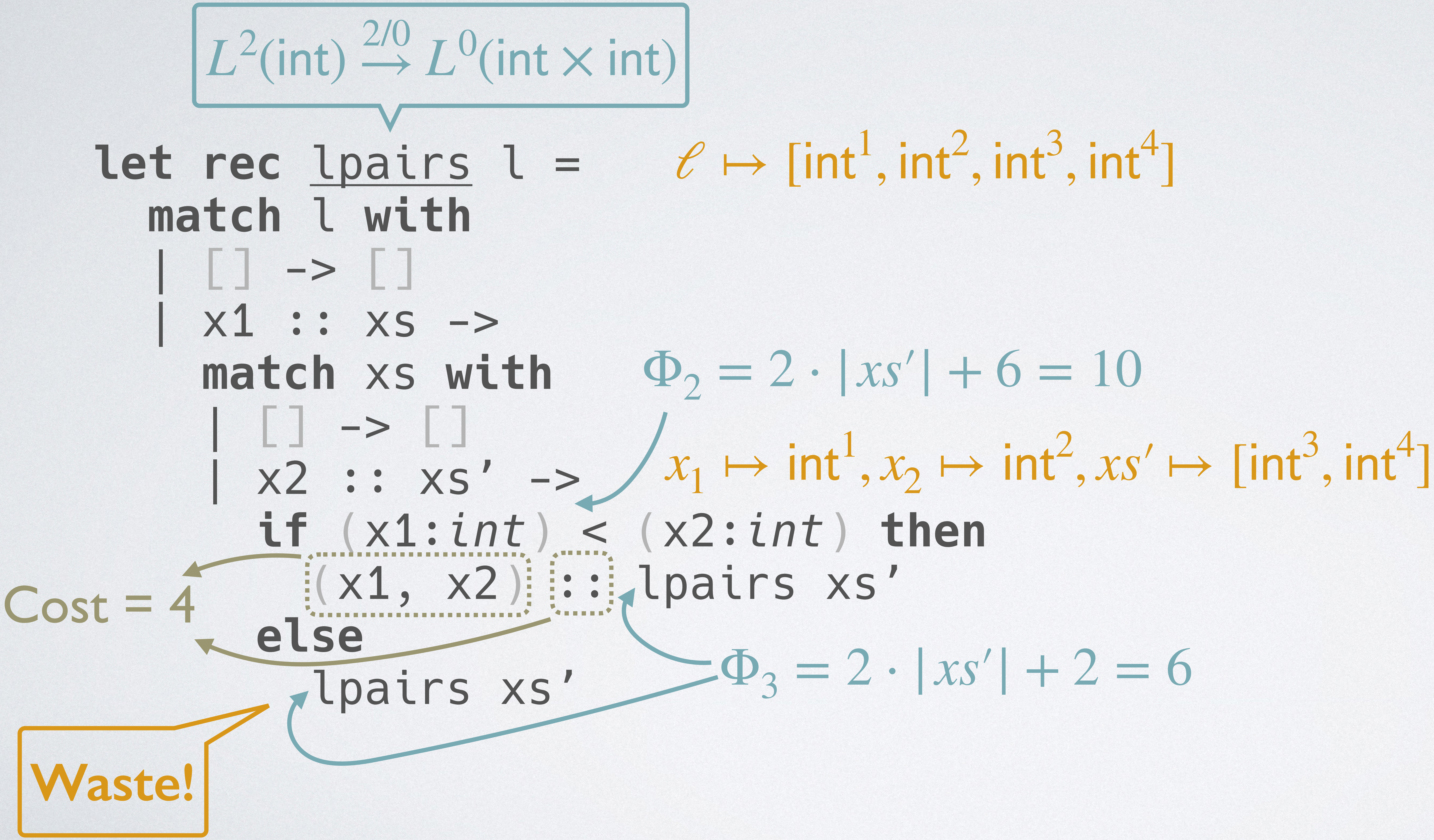

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If an execution path does not have **potential waste**, it must expose the worst-case resource usage.

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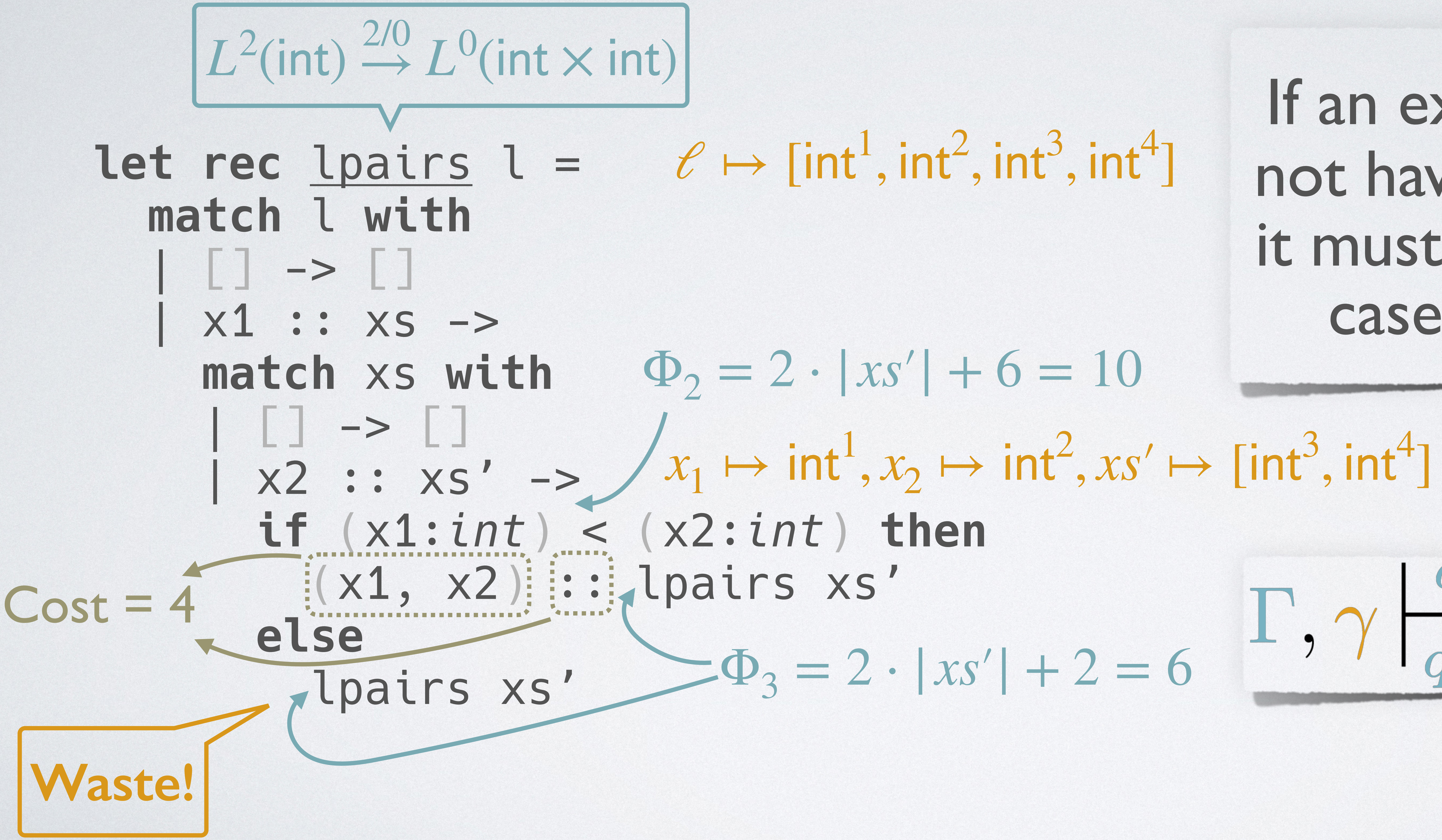

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- **Relative completeness**: If there is an input of some given shape that causes the program to consume ***exactly*** the same amount of resource as the inferred ***upper bound*** (by RaML), then the algorithm is able to find ***a corresponding execution path***.

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$$\boxed{\text{Then}}\quad \frac{\gamma \vdash e_1 \Rightarrow \langle \psi, S \rangle}{\gamma \vdash \mathsf{if}\ e\ \mathsf{then}\ e_1\ \mathsf{else}\ e_2 \Rightarrow \langle \gamma(e) \wedge \psi, S \rangle}$$

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- How about **eliminating** some generation rules?

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Still Sound!

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Still Sound!

- **Generalization**: enforce all the calls with the ***same*** shape of inputs execute the ***same*** path in the function body.

Overview

- ☑ Motivation
- ☑ Resource Aware ML (RaML)
- ☑ Type-Guided Worst-Case Input Generation
- □ Evaluation

Implementation

- We implemented the generation algorithm for a purely functional fragment of Resource Aware ML (RaML), including **higher-order functions, user-defined data structures**, and **polynomial resource bounds**.

- We used the off-the-shelf SMT solver Z3.

Benchmarks (Selected)

Description	Shape	ALG	ALG+H1	ALG+H2
Insertion sort	200 integers	7.74s	**6.97s**	94.81s
Quicksort	200 integers	T/O	**53.23s**	157.21s
Lexicographic quicksort	Lists of length 100, 99, …, 1	439.35s	**438.79s**	T/O
Functional queue	200 operations	**444.64s**	T/O	T/O
Zigzag on a tree	200 internal nodes	T/O	T/O	**4.87s**
Hash table for 8-char strings	64 insertions	7.64s	**7.62s**	181.74s

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EXAMPLE: HASH TABLE

- Customized resource metric: count for **hash collisions**
- Use a hash function from a vulnerable PHP implementation
- The program inserts 64 strings into an empty hash table
- Our algorithm “**realizes**” that it should find 64 strings with the same hash key, in order to trigger the most collisions

Summary

Type-Guided Symbolic Execution for Worst-Case Input Generation

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- Formally developed algorithm
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Theoretical Results

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Theoretical Results

- Integrated with RaML
- Effective on 22 benchmark programs

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Limitations:

- Purely functional programs
- Only work for tight bounds
- Depend on RaML

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Limitations:

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Future work:

- Support side effects
- Interact with resource analysis
- General theory for worst-case analysis

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